Informational and/or Transactional Websites: Strategic Choices in a Distribution Channel

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Abstract

While most businesses have faced the decision of whether to operate an informational and/or a transactional website, the literature on website selection in marketing channels remains very sparse. This paper proposes an analytical framework that compares scenarios where a manufacturer uses either an informational, a transactional, or both transactional and informational website in a distribution channel formed by one manufacturer and one retailer. We find that the selection of the optimal website depends on the online market base of the product, the effectiveness of the manufacturer-controlled online communications, and the cross-price effect between online and offline channels. For both the

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manufacturer and retailer, informational websites are preferable when the online market base is small. With larger online markets, the manufacturer may prefer either informational and transactional websites or exclusively informational websites, while the retailer is always better off with an exclusively informational website. Theoretical and managerial implications of these findings are discussed.

**Keywords**: Marketing strategies, multichannel commerce, distribution channels, game theory.
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Abstract

While most businesses have faced the decision of whether to operate an informational and/or a transactional website, the literature on website selection in marketing channels remains very sparse. This paper proposes an analytical framework that compares scenarios where a manufacturer uses either an informational, a transactional, or both transactional and informational website in a distribution channel formed by one manufacturer and one retailer. We find that the selection of the optimal website depends on the online market base of the product, the effectiveness of the manufacturer-controlled online communications, and the cross-price effect between online and offline channels. For both the manufacturer and retailer, informational websites are preferable when the online market base is small. With larger online markets, the manufacturer may prefer either informational and transactional websites or exclusively informational websites, while the retailer is always better off with an exclusively informational website. Theoretical and managerial implications of these findings are discussed.

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1 Introduction

A crucial issue facing marketing managers when developing their digital channel strategy is whether to develop an informational or a transactional website (Lee and Grewal, 2004). An informational website is a support tool designed to offer information to various stakeholders, including consumers, about products, services, and all relevant aspects of the business; a transactional website offers the possibility for consumers to purchase products online (Van Nierop et al., 2011). Although it is believed that these two types of websites are developed to improve the company’s performance, their strategic implications as well as their impacts on channel members’ decisions and profits are different. Because transactional websites are alternative distribution channels, they change the nature of vertical interactions between manufacturers and their traditional offline retailers. Generally, manufacturers that sell their products to consumers online are not only partners with their offline retailers, but also compete with them to a certain extent (Biyalogorsky and Naik, 2003; Chiang et al., 2003; Neslin et al., 2006; Pu et al., 2017; Yan and Pei, 2011, 2015; Yoo and Lee, 2011). On the other hand, informational websites contribute to expanding consumer knowledge about the product and promotional offers with the goal of increasing offline sales at the retail level, avoiding competition with traditional retailers (Van Nierop et al., 2011).

Choosing between a transactional and informational website is an important managerial issue that virtually all major companies have faced or will face. Casual observations and published works provide many interesting examples. Almost two decades ago, Levi Strauss & Co. started a transactional website that was soon turned into
an exclusive informational website due to conflicts with offline retailers (Chiang et al., 2003; Yan, 2011). Today, this company operates a website that is both transac-
tional and informational. Gibson Musical Instruments (GMI) stopped online sales
operations just one month after their start to content dealers (Lee et al., 2003).
Since then, GMI has operated a state-of-the-art informational website. Zara first
operated an informational website that was later upgraded to a transactional and
informational website (Lee and Grewal, 2004; Van Nierop et al., 2011). On the other
hand, The Gap is reported to have launched, in 1996, a website that was transac-
tional and informational (Lee and Grewal, 2004). These few examples raise several
unanswered questions, including: Why do manufacturers operate a specific type of
website? Why, over time, do some manufacturers change or upgrade the type of
websites they operate? Clearly, there is a need for a comprehensive framework that
can help answer these questions and also guide the selection of the type of activities
that manufacturers can perform online.

The purpose of this paper is to analytically investigate, in a two-member channel,
the conditions under which the manufacturer should offer either an informational, a
transactional, or both an informational and transactional website. To reach this goal,
we consider a situation where the manufacturer and retailer in a bilateral monopoly
play three games: Games I, T, and TI. Game I is played when the manufacturer sup-
ports the retailer’s offline activities by providing online communications that stimu-
late offline sales. In Game T, the manufacturer develops a transactional website that
competes with the retailer’s traditional offline offering. Game TI is the combination
of the first two games, in which the manufacturer develops a transactional website
and also offers online marketing communications to stimulate sales. In each of the three games, the manufacturer and retailer set their pricing decisions to maximize their own profits. We identify the conditions under which each of these three games can be played and compare the players’ payoffs to establish their preferences. We show that, under certain conditions, the two channel members can agree to play either Game I or Game TI.

To the best of our knowledge, this paper is the first analytical attempt to formally investigate the role manufacturers’ own websites can play in their channel strategy. The premise of our work is that the addition of either type of website influences the performance of channel members. Operating a transactional website corresponds to an online market expansion strategy, which allows manufacturers to reach consumers who will not purchase otherwise, while the adoption of an informational website refers to a penetration strategy via online communications that aims at increasing offline sales. In particular, this paper builds on previous empirical and analytical works which are discussed below.

Research on the use of transactional websites in marketing channels has mainly investigated whether such sites increase overall sales or cannibalize offline sales, with the idea that cannibalization is negative for firm performance. For instance, using the data from Tower Records, an offline music retailer that expanded to online retailing, Biyalogorsky and Naik (2003) find that online sales operations do not significantly cannibalize offline sales, but contribute to building online equity that positively affects future online sales. Deleersnyder et al. (2002) use data from 85 Internet channel additions over 10 years in newspaper industries in the UK and The Netherlands and
conclude that cannibalization of offline sales is minimal, unless online channels closely mimic the positioning of offline channels. Geyskens et al. (2002) study the addition of a transactional website in the newspaper sector and find both positive and negative effects on the company’s stock prices. Wolk and Skiera (2009) use survey data from multichannel retailers and find that transactional websites fail if they are not differentiated from offline channels, but their net effect on performance is positive, with a stronger positive effect on strategic than on financial performances. A more recent related study by Waterman and Ji (2012) on ten U.S. major media categories over six decades reports that increasing revenues from Internet distribution are exceeded by declines in revenues from offline channels. This work supports the view that not only are the total market sizes of these media declining, but cannibalization does occur at the expense of traditional offline channels as well.

A few empirical works investigate the impact of informational websites on firm performance. For example, Lee and Grewal (2004) study the financial market valuation of 106 firms over 9 years and find that markets reward firms for developing informational websites. Pauwels et al. (2011) consider the case of a traditional offline department store with various product categories and study the revenue implications of the introduction of an informational website. They find, among others, that the impact of the introduction of an informational website depends both on the product category and on the customer segment. In particular, the revenue impact of an informational website is higher for sensory products than for non-sensory products, and it is also higher for consumers who live far away from the store.

On the other hand, several analytical works have studied the addition of a trans-
actional website in the context where the manufacturer already sells the products to offline retailers. These works are strategic in nature as they consider the interactions between channel members and establish the conditions under which manufacturers can operate dual channels. Channel decisions are made so as to help channel members keep or improve their individual performance, and to avoid or lessen channel conflicts compared to situations where manufacturers deal exclusively with offline retailers (e.g., Chiang et al., 2003; Yan, 2011; Yan and Pei, 2011; Yao and Liu, 2005). The introduction of a transactional website by a manufacturer in such a context is known to mitigate the double marginalization effect, increase market coverage, generate price competition, and potentially lead to price discrimination to better serve both online and offline consumers (Yoo and Lee, 2011). The effect of channel cannibalization is believed to be minimal due to the fact that more channels serve a wider variety of consumers.

The main innovation of this paper, compared to previous analytical works, is the recognition of the importance of informational websites in marketing channels. Until now, analytical works have been limited to the introduction of transactional websites in conventional channels where manufacturers deal exclusively with retailers without any other online presence (e.g., Cheng and Xiong, 2015; Yan, 2011; Yan and Pei, 2011; Yoo and Lee, 2011). While situations where manufacturers do not provide commercial information online from their own websites to stimulate sales either online or offline do exist, they are increasingly very scarce. The real challenge for manufacturers may not be choosing between selling or not selling online as in the current analytical literature, but finding the right mix of commercial activities
that can be performed online to improve channel efficiency. In any case, research on the effects of informational websites on firm performance discussed above and on the effects of online information on consumer behavior (e.g., Kulkarni et al., 2012; Van Nierop et al., 2011; Ratchford et al., 2007; Viswanathan et al., 2007) indicates that online controlled-information by manufacturers should not be ignored in strategic interactions with retailers.

The remainder of the paper is organized as follows. Section 2 describes the three models. Section 3 describes the derivation of the equilibrium solutions and discusses their feasibility conditions. Section 4 discusses the players’ preferences. Finally, Section 5 concludes.

2 Models

Consider a typical two-member channel in which a manufacturer sells a product to a retailer, who then sells it to consumers offline or in a physical store. We assume that there is either no external competition at the manufacturing and retail levels, or if competition does exist, it does not change vertical interactions between the two channel partners. The manufacturer contemplates the possibility of developing a website that can help to increase her profits and, if possible, also enhance the profitability of the retailer. The manufacturer has to decide whether to use the website for informational, transactional, or for both informational and transactional purposes. These three situations correspond to three different games as described below.
2.1 Game I: Informational website

In this game, the manufacturer opts for an exclusively informational website that supports offline sales. The manufacturer sets the wholesale price, $w$, for the product and the level of communication effort, $i$, to provide through the website. The retailer sets the retail price, $p_f$, consumers pay for the product. We assume that the demand for the product at the retailer’s store, $q_f$, linearly depends on the retail price and the information provided online by the manufacturer. For simplicity, we adopt the following demand function:

$$q_f = a - p_f + ci.$$  

The parameter $a$ is positive and represents the baseline demand or total market base of the product. The parameter $c \in (0,1)$ denotes the effectiveness of online communications undertaken by the manufacturer from her own website on retail sales. Other forms of online communications (e.g., social media efforts) that are either controlled or not controlled by the manufacturer are not considered in this research. The parameter $c$ depends on several factors that we do not control in this research, including the type of product sold, consumer behavior, and the nature of commercial information provided by the manufacturer. For example, for some product categories, promotional information may be more effective in raising sales than information that aims at increasing consumer knowledge about the product. On the other hand, we normalize the price sensitivity to 1 to focus on other parameters, critical to reaching the goal of this research.

As it is common in the marketing literature, production, administrative, and operational costs, including the costs of developing and maintaining the website,
are set to zero. The manufacturer’s costs of delivering updated marketing communications through the website are quadratic and given by $i^2$. Let $M$ and $R$ denote, respectively, the profit functions of the manufacturer and retailer. The two channel members set their decision variables so as to maximize their following profit functions:

$$M = w[\alpha - p_f + ci] - i^2,$$

$$R = (p_f - w)[\alpha - p_f + ci].$$

### 2.2 Game T: Transactional website

In this game, the manufacturer opts for an exclusively transactional website. While the product continues to be available offline through the retail store, it is also sold online by the manufacturer. The exclusively transactional website offers nothing that can stimulate the retailer’s offline sales. This configuration is the most studied in the current analytical literature (e.g., Pu et al., 2017; Yan, 2011; Yoo and Lee, 2011). In this case, in addition to setting a wholesale price as in the previous game, the manufacturer also sets a retail price, $p_{mo}$, for her online offering. The pay-to-order payment scheme is adopted for the manufacturer’s online operations, as the online retail price includes delivery fees (Xu et al., 2017). The retailer also still sets a retail price for those who want to purchase at his store. The demand functions for the manufacturer’s online, $q_{mo}$, and retailer’s offline, $q_f$, demands are given by:

$$q_{mo} = o\alpha - p_{mo} + b(p_f - p_{mo}),$$

$$q_f = (1 - o)\alpha - p_f + b(p_{mo} - p_f).$$
The parameter \( o \in (0, 1) \) denotes the proportion of the online baseline demand, while \( (1 - o) \) represents the proportion of the offline baseline demand. As in the first model, the parameter \( a \) is still the total market base of the product, which is now divided into two groups of consumers: those who prefer to get the product online and those who prefer to get it offline free of charge (Yan, 2011). The new parameter \( b \in (0, 1) \) represents the cross-price effect or the intensity of price competition between offline and online offerings. If \( b = 0 \), there is no price competition between the two channels and people who buy from one channel do not care about the price offered in the other channel. If \( b = 1 \), any difference between offline and online prices will heavily impact the demands of the two channels. Many factors may affect this parameter, including the differentiation of offline and online offerings, and consumer attachment to channels. For parsimony, we assume a symmetric cross-price effect between online and offline channels in this case.

In addition, we assume that the manufacturer’s online retail price is higher than her wholesale price to the offline retailer (i.e., \( p_{mo} > w \)). This is a practical assumption that ensures the retailer does not have an incentive to buy online for offline reselling.

As in the previous model, production, administrative, and operational costs, including the costs of developing and maintaining the website, are set to zero. The profit functions of the two channel members are:

\[
M = wq_f + p_{mo}q_{mo},
\]

\[
R = (p_f - w)q_f.
\]
Compared to Game I, the manufacturer now generates revenues from both online and offline channels, while the retailer’s only revenues come from the offline channel.

2.3 Game TI: Transactional and informational website

We now consider the scenario where the manufacturer develops a website for both transactional and informational purposes in a more integrated approach. The goal is to take advantage of the full potential of one’s own website as a communication tool and as a distribution channel. As a communication tool, the manufacturer uses the website to perform activities that support and stimulate both online and offline sales. The retailer still determines a retail price, while the manufacturer now sets a retail price for the online offering, a wholesale price for the offline offering, and the online communication effort to consumers. The online and offline demand functions depend on the retail prices and the manufacturer’s online communication effort as follows:

\[ q_{mo} = a - p_{mo} + b(p_f - p_{mo}) + ci, \]

\[ q_f = (1 - o)a - p_f + b(p_{mo} - p_f) + ci. \]

Most of the assumptions are identical to those previously discussed. An additional assumption here is that the information provided on the website is equally effective (c) on the online and offline demands. This is obviously a simplification given that online and offline buyers may not react identically to online communication activities. In any case, the manufacturer adopts an integrated communication approach that reaches both online and offline buyers with the same message. For example, when
online promotional activities are undertaken, they are accessible to both online and offline buyers. Consumers are given the choice to use their preferred channel, but there is no communication effort from the manufacturer to change their preference from one channel to another.

The profit functions of the two channel members in this game are as follows:

\[ M = wq_f + p_{mo}q_{mo} - t^2, \]

\[ R = (p_f - w)q_f. \]

Table 1 summarizes our specifications for the three models.

Insert Table 1 about here

3 Equilibrium solutions and feasibility conditions

The manufacturer and retailer play a Stackelberg game in the indirect channel in each of the three games. The manufacturer is assigned the leadership role and the retailer is the follower in the indirect channel. The sequence of moves for the two players in each game is as follows. When applicable, the manufacturer first announces her wholesale price and the level of communication effort and the retailer and the manufacturer (Models T and TI) simultaneously set their retail prices.

Optimal solutions are derived backwards. This means that the retailer in Model I and the retailer and manufacturer in Models T and TI first obtain their optimal retail prices. The manufacturer’s optimal wholesale prices and online communication efforts are derived after the introduction of the optimal retail prices into the
manufacturer’s profit functions. The derivation and the equilibrium solutions for the three games are detailed in the Appendix. As well, in the Appendix, we further set the market base \( (a) \) to 1 without loss of generality. All concavity conditions for the manufacturer’s and retailer’s problems in each game are verified. We obtain a unique equilibrium solution for each of the three games. These optimal solutions all depend on the model parameters. We therefore investigate whether some conditions need to be imposed on the parameters to obtain non-negative profits, positive pricing decisions and margins, and positive online investments.

Game I is always feasible regardless of the values of the parameter \( c \) in the range of \((0, 1)\). To identify the conditions under which Games T and TI are feasible, we conduct numerical simulations for different values of the parameters. Our findings are summarized in Figure 1 below.

Insert Figure 1 about here

Figure 1.1 displays the feasible conditions of Game T in the space of the two relevant parameters in this case: \( o \) and \( b \). UF denotes the area in the parameter space in which the equilibrium solution is unfeasible. This game is always feasible when the online market base is at least identical to the market base at the retailer’s store. If the online market base is relatively smaller than the offline market base \((0.4 < o < 0.5)\), the intensity of price competition between the two channels does matter. Game T is more likely to be feasible in this range if the intensity of price competition between the two channels is very high.

For Game TI, there are three relevant parameters: \( o, b, \) and \( c \). We find that, regardless of the values of the other model parameters, the equilibrium solution of
this game is always unfeasible (feasible) when \( o < 0.4 \) \((o > 0.5)\). On the other hand, when, \( 0.4 < o < 0.5 \), the equilibrium solution of this game can be feasible depending on the values of the two other model parameters, \( c \) and \( b \). Figure 1.2 provides an illustration. In particular, Game TI is more likely to be feasible in this area when higher values of the effectiveness of the manufacturer’s online communications are combined with higher levels of price competition between channels.

4 Choosing a website strategy

In this section, we compare the equilibrium strategies and profits of the three games taking into account the feasibility conditions previously discussed. In some cases, we are able to obtain analytical results. In others, we resort to numerical analyses to obtain meaningful insights. In particular, we vary the parameters values in the following ranges: \( b, c, o \in (0, 1) \) by a step size of 0.02, in the feasible domain where the three games can be effectively played. All results are summarized in Table 2. The columns \( T - I \), \( TI - T \), and \( TI - I \) represent the findings of the comparisons between the two specified games. The sign "+" ("−") indicates that we are able to prove that the outcome of the first game is superior (inferior) to the outcome of the second game. The sign "±" indicates that we are able to find values of the parameters for which any of the two games can have an inferior or superior outcome depending on the values of the parameters. All proofs are in the Appendix.

Insert Table 2 about here
4.1 Informational website

The proposition below indicates when the manufacturer should automatically opt for an exclusively informational website.

Proposition 1 When the online market base is not very large \((o < 0.4)\), the manufacturer should have an exclusively informational website to support offline sales.

This finding is derived directly from the analysis of the feasibility conditions of the three games. Games T and TI are not feasible when \(o < 0.4\), while Game I is always feasible. Consequently, opening a transactional website when the online market base is not large enough is not conducive to sustainable business practices between the manufacturer and retailer. In particular, because the online market base is relatively small, the manufacturer should set a relatively small online price that contributes to the cannibalization of the offline sales. This situation is known to create channel conflicts between manufacturers and retailers as it directly damages the profitability of retailers’ offline stores. As a matter of fact, it is reported that Levi Strauss & Co. originally terminated its online sales operations due to such conflicts. After this decision, this company did, however, maintain for many years an informational website that provided useful information to various stakeholders, including customers. A company such as Zara had an informational website for some time before upgrading to an informational and transactional website.
4.2 Different preferences

Considering the feasibility conditions of the three games, results derived from the comparisons of the manufacturer’s profits are summarized in the next proposition.

**Proposition 2** When the three games are simultaneously feasible, the manufacturer: (1) prefers a website used for both informational and transactional purposes to an exclusively transactional website, (2) may prefer either an exclusively transactional website or an exclusively informational website depending on the parameters, and (3) may prefer either a transactional and informational website or an exclusively informational website depending on the parameters.

Given that the manufacturer always prefers an informational and transactional website to an exclusively transactional website in Proposition 2, the real decision for the manufacturer is therefore whether to develop an exclusively informational website or a website that combines both transactional and informational features. We illustrate in Figure 2 below how the manufacturer’s profits compare in these two situations.

Insert Figure 2 about here

The findings in Figure 2 support the view that pursuing a strategy that combines market expansion and market penetration is more rewarding for the manufacturer than further penetrating the offline market when the online market becomes very large, given the other model parameters. This is because an informational and transactional website allows the manufacturer to also serve new customers that
the retailer’s store cannot reach even with the support of an informational website. The manufacturer’s maximum sales are then achieved by using the Internet to further penetrate the offline market and to expand sales online. This finding is consistent with previous works that support the view that serving a broader base of consumers benefits the manufacturer (e.g., Yoo and Lee, 2011). However, when the manufacturer’s online communication is not very effective, pursuing an offline market penetration strategy alone with online communications can still be the best strategic choice for relatively large online markets. This ensures that all consumers who look for information online and can buy offline are well served before going after those who exclusively purchase online.

**Proposition 3** When the three games are simultaneously feasible, the retailer prefers:

1. an exclusively informational website to an exclusively transactional website,
2. an exclusively informational website to an informational and transactional website, and
3. an informational and transactional website to an exclusively transactional website.

As expected, the manufacturer’s exclusive offline penetration strategy that consists of stimulating only offline sales through online communications better serves the interests of the retailer than any other of the two strategies where the manufacturer also starts selling directly online. As we can see in Table 2, this finding is explained by the fact that the manufacturer’s online expansion introduces competition in the market. As a consequence, the retailer reduces her offline retail price to stay competitive and also loses some of her customers to the manufacturer’s online channel due to the well-known cannibalization effect (Biyalogorsky and Naik, 2003;
Geyskens et al., 2002). Even if the manufacturer’s wholesale price goes down when a transactional component is added to the website, such a change does not generally aim at increasing the retailer’s profitability (Yoo and Lee, 2011). The manufacturer is more concerned with maximizing her two revenue streams in Games T and TI than helping the retailer to reach the maximum sales possible as in Game I.

The comparison of the retailer’s profits in Games TI and T shows that the retailer is always better off in Game TI. This is essentially due to the fact that the manufacturer commits more resources to providing online information that benefits her through both the online and offline channels. As a consequence, sales via the two channels increase, allowing all channel members to charge higher prices and obtain better returns.

5 Conclusion

This paper analytically examines the type of website a manufacturer should develop to improve channel performance when dealing with an offline retailer in the context of a bilateral monopoly. The following three types of websites are considered: An informational website that supports offline sales with online communications, a transactional website that allows the purchase of the product online, and an integrated website that performs both of the functions of the first two websites. The optimal equilibrium strategies and profits are obtained for these three games. Conditions under which these equilibrium solutions are feasible are identified. The comparisons of the players’ profits across these games reveal their preferences with respect to the
different websites. The theoretical and managerial implications of our findings are discussed below.

First, we find that when the offline (online) market base is relatively large (small), the manufacturer should exclusively operate an informational website to further penetrate the offline market. The two other website alternatives should only be considered when the online market base is significant enough to allow sustainable business between channel members. Otherwise, further penetration of the offline market with the exclusive use of online marketing communications is desirable. Therefore, factors that affect the size of the online market should drive any online expansion decision. Examples of such factors include product compatibility with online sales, customer dispersion, and customer aversion to or preference for online shopping. The existence of a relatively large online market may not, however, be enough to support the development of a transactional website. In some cases, the manufacturer should also consider the effectiveness of her online communications directed to offline consumers and the cross-price effect between online and offline channels. As a result, even if having a transactional website is conducive to a sustainable business for all channel members, a manufacturer can still adopt an informational website as the best profit-maximizing alternative.

Second, the manufacturer and retailer earn more profits when the manufacturer operates both an informational and transactional website than an exclusively transactional website. To understand the importance of this finding, one has to remember that the two channel members’ preferences may differ when exclusively informational and transactional websites are considered. This means that not only is the manu-
facturer financially better off, but she also avoids potential channel conflict with the retailer by adopting a transactional and informational website instead of an exclusively transactional one. Therefore, from a retailer’s perspective, the common argument that manufacturers use transactional websites to target consumers who do not normally purchase offline is more convincing if these websites are developed to support both online and offline sales with online integrated marketing communications programs. An online market expansion strategy should not be undertaken at the expense of a penetration strategy that appeals to people who look for information online and make their purchases offline.

Third, while the manufacturer may be better off with an informational and transactional website, it does not serve the best interest of the retailer compared to an exclusively informational website. In this case, the manufacturer’s preference depends on the offline market base, the effectiveness of her online communications, and the price competition between online and offline channels. This finding suggests that manufacturers who first start informational websites may face opposition to their online expansion ambitions from retailers if market conditions change to allow them to add a transactional component to their websites. Sales cannibalization, unfair price competition, and lack of effective online communications that support offline sales may all be used by retailers to advocate for the status quo, which is maintaining the use of an exclusively informational website. In such a context, the onus is on the manufacturer to demonstrate that her online expansion overcomes these legitimate concerns.

Summarizing, the theory developed in this paper supports the view that manu-
facturers’ observed website practices can be explained by the combination of three factors: the online (offline) market base, the effectiveness of manufacturer-controlled online communications, and the intensity of price competition between online and offline channels. For example, the reported increase of online shopping trends changes the online market base for several products. Everything else being equal, this might explain why in the 90s, Levi Strauss & Co. could not convince its dealers to accept its online sales operations, and other less adventurous companies opted to start with informational websites. Given the nature of its products, Gibson Musical Instruments has not yet reached the online market base threshold necessary to start successful online sales operations that will not jeopardize offline operations. One may imagine that once this threshold is reached, expanding online could even become beneficial to dealers. As a matter of fact, Geyskens et al. (2002) reported that Barnes and Noble saw a record sales increase in offline stores upon launching online sales operations.

Future research can extend this work in many ways. We have kept our model simple to derive meaningful analytical insights. Future studies can explore more complex set-ups such as ones where competition from other manufacturers and online retailers is considered. Other extensions can also be explored such as the operational costs of managing the website, especially for transactional purposes, and the retailer’s promotional and assortment decisions that might affect the success of the online channel.
References


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6 Appendix

6.1 Equilibrium solutions and feasibility conditions

We solve each game by backward induction to get the equilibrium solution for each game. We get the following results.
6.1.1 Game I

**Proposition 4** In Game I, the equilibrium strategies and profits are as follows

\[ w^I = \frac{4}{8-c^2}, \quad \hat{v}^I = \frac{c}{8-c^2}, \quad p^I_f = \frac{6}{8-c^2}, \]

\[ R^I = \frac{4}{(8-c^2)^2}, \quad M^I = \frac{1}{8-c^2}. \]

**Proof:** We solve Game I by backward induction. We start by solving the retailer’s and the manufacturer’s problems simultaneously given by

\[ \max_{p_f} R = (p_f - w) [a - p_f + ci], \]

\[ \max_{p_{mo}} M = w [a - p_f + ci] - i^2, \]

and get the reaction functions by solving the system of first-order optimality conditions \( \frac{\partial R}{\partial p_f} = \frac{\partial M}{\partial p_{mo}} = 0 \). The concavity conditions are verified for any \( c \in (0, 1) \). The obtained reaction functions are then inserted into the manufacturer’s problem given by

\[ \max_{i, w} M = w [a - p_f + ci] - i^2. \]

We get the equilibrium manufacturer’s online and wholesale prices by solving her first-order optimality conditions \( \frac{\partial M}{\partial r} = \frac{\partial M}{\partial w} = 0 \). The manufacturer’s concavity conditions are verified for any \( c \in (0, 1) \). The obtained expressions are then inserted into the reaction functions to get the equilibrium solution as a function of the model parameters given in proposition 4.
Note that the equilibrium solution satisfies the conditions that all equilibrium strategies, as well as the retailer’s margin, demand and profits are positive for any $c \in (0, 1)$.

## 6.1.2 Game T

### Proposition 5

In Game T, the equilibrium strategies and profits are as follows

\[ w^T = \frac{9b^4 + (32 - 7o)b^3 + 24(2-o)b^2 + 8(4-3o)b - 8(o-1)}{(36b^4 + 118b^3 + 146b^2 + 80b + 16)}, \]  
\[ p^T_f = \frac{(b + 2)(3b + 2)N_1}{(108b^6 + 642b^5 + 1526b^4 + 1880b^3 + 1272b^2 + 448b + 64)}, \]  
\[ p^T_{mo} = \frac{(b + 2)(9b^3 + 5b^2 + 20b^2 + 14b + 10b + 8o)(3b + 2)(1 + b)}{(108b^6 + 642b^5 + 1526b^4 + 1880b^3 + 1272b^2 + 448b + 64)}, \]  
\[ q^T_f = \frac{(1 - o)(3b^2 + 4b + 2)(b + 2)(1 + 2b)(3b + 2)(1 + b)}{(54b^6 + 321b^5 + 763b^4 + 940b^3 + 636b^2 + 224b + 32)}, \]  
\[ q^T_{mo} = \frac{(b + 2)(6b^3 + 3b^3 + 21b^2 + 2b + 2b + 2b + 8o)(1 + 2b)(3b + 2)}{(108b^6 + 642b^5 + 1526b^4 + 1880b^3 + 1272b^2 + 448b + 64)}, \]  
\[ R^T = \frac{(o - 1)^2 (3b^2 + 4b + 2)^2}{(9b^2 + 16b + 8)^2 (1 + b)}, \]  
\[ M^T = \frac{N_2}{4(2b + 1)(b + 1)(16b + 9b^2 + 8)}, \]

where

\[ N_1 = (9b^4 - 19b^3o + 44b^3 - 46b^2o + 70b^2 - 40bo + 48b - 12o + 12), \]  
\[ N_2 = 9b^4 + 2(4o^2 + o + 12)b^3 + (29o^2 + 28 - 8o)b^2 + 16(2o^2 + 1 - o)b + 4(1 - o)^2 + 4o^2. \]
Proof: We solve Game T by backward induction. We start in stage 2 by solving the retailer’s and the manufacturer’s problems simultaneously given by

\[
\begin{align*}
\max_{p_f} R & = (p_f - w) [(1 - o)a - p_f + b(p_{mo} - p_f)] , \\
\max_{p_{mo}} M & = w [(1 - o)a - p_f + cs] + p_{mo} [(oa - p_{mo} + b(p_f - p_{mo})],
\end{align*}
\]

and get the price reaction functions \( p^{RF(T)}_f \) and \( p^{RF(T)}_{mo} \) by solving the system of first-order optimality conditions \( \frac{\partial R}{\partial p_f} = \frac{\partial M}{\partial p_{mo}} = 0 \). The retailer’s and manufacturer’s concavity conditions are negative and given by \(-2 - 2b\), and are therefore verified for all parameters \( o, b \in (0, 1) \). These reaction functions are given by

\[
\begin{align*}
p^{RF(T)}_f & = \frac{(3b^2 + 4b + 2) w + 2(1 - o) + b(2 - o)}{(b + 2)(3b + 2)}, \\
p^{RF(T)}_{mo} & = \frac{3b(b + 1) w + b + bo + 2o}{(b + 2)(3b + 2)}.
\end{align*}
\]

The prices \( p_{mo} \) and \( p_f \) are then replaced by the obtained reaction functions \( p^{RF(T)}_f \) and \( p^{RF(T)}_{mo} \) in the manufacturer’s problem in stage 1, which are given by

\[
\begin{align*}
\max_{w} M & = w \left[ (1 - o)a - p^{RF(T)}_f + b(p^{RF(T)}_{mo} - p^{RF(T)}_f) \right] \\
& + p^{RF(T)}_{mo} \left[ ao - p^{RF(T)}_{mo} + b(p^{RF(T)}_f - p^{RF(T)}_{mo}) \right].
\end{align*}
\]

We get the manufacturer’s equilibrium wholesale price by solving her first-order optimality condition \( \frac{\partial M}{\partial w} = 0 \). The manufacturer’s concavity condition is negative.
and given by
\[-\frac{2(1 + 2b) (b + 1) (9b^2 + 16 + 8)}{(b + 2)^2 (3b + 2)^2}.
\]

Therefore, the manufacturer’s concavity condition is verified for all parameters \(o, b \in (0, 1)\). The obtained expressions are then inserted into the reaction functions to get the equilibrium solution as function of the model parameters given in proposition 5.

Note that, for any \(o, b \in (0, 1)\), it is easy to see from the expressions in (1)-(7), that the equilibrium decisions and output are always positive. Additionally, we verify that the retailer’s margin given by
\[(p_f^T - w^T) = \frac{(1 - o) (4b + 3b^2 + 2)}{(b + 1) (16b + 9b^2 + 8)}\]
is also positive for any \(o, b \in (0, 1)\). Finally,
\[(p_{mo}^T - w^T) = \frac{(b + 2) (19b + 12b^2 + 8) o - (3b^3 + 18b^2 + 22b + 8)}{2 (2b + 1) (b + 1) (16b + 9b^2 + 8)}.
\]

Hence, the condition that guarantees a manufacturer’s online price that is higher than its wholesale price is as follows:
\[o > \frac{3b^3 + 18b^2 + 22b + 8}{(b + 2) (19b + 12b^2 + 8)}.
\]

Therefore, the above condition defines the feasibility domain for Game T (represented in Figure 1.1). Since the right-hand expression in (8) is strictly increasing in \(b\), in all the feasible domain of Game T, the parameter \(o\) will exceed the maximum
value of this expression, which is realized for \( b = 1 \) and is given by 0.4359.

### 6.1.3 Game TI

**Proposition 6** In Game TI, the equilibrium strategies are denoted by the superscript \((TI)\) and are as follows

\[
w^{TI} = \frac{2X_1}{(1 + 2b)(18b^3c^2 + 45b^2c^2 - 36b^3 + 40bc^2 - 100b^2 + 12c^2 - 96b - 32)}, \tag{9}
\]

\[
i^{TI} = \frac{-c(9b^3 + 5b^2 o + 20b^2 + 8bo + 16b + 4o + 4)}{(18b^3c^2 + 45b^2c^2 - 36b^3 + 40bc^2 - 100b^2 + 12c^2 - 96b - 32)}, \tag{10}
\]

\[
p^T_{f} = \frac{X_2}{(1 + 2b)(18b^3c^2 + 45b^2c^2 - 36b^3 + 40bc^2 - 100b^2 + 12c^2 - 96b - 32)}, \tag{11}
\]

\[
p^{T}_{m} = \frac{X_3}{(1 + 2b)(18b^3c^2 + 45b^2c^2 - 36b^3 + 40bc^2 - 100b^2 + 12c^2 - 96b - 32)}, \tag{12}
\]

\[
R^{TI} = \frac{(2c^2 o - c^2 - 4o + 4)^2(3b^2 + 4b + 2)^2(1 + b)}{(18b^3c^2 + 45b^2c^2 - 36b^3 + 40bc^2 - 100b^2 + 12c^2 - 96b - 32)^2}, \tag{13}
\]

\[
M^{TI} = \frac{X_3}{(1 + 2b)(18b^3c^2 + 45b^2c^2 - 36b^3 + 40bc^2 - 100b^2 + 12c^2 - 96b - 32)^2}. \tag{14}
\]

where

\[
X_1 = 8(o - 1) + b (-48b + 24o + 7b^2 o + 24bo - 32b^2 - 32 - 9b^3)
\]
\[+ (3b + 2) (1 - 2o)((b + 1) c)^2, \]

\[
X_2 = 24(o - 1) - 18b^4 + (38o - 88) b^3 + (92o - 140) b^2 + (80o - 96) b
\]
\[+ c^2 (22b + 27b^2 + 12b^3 + 6) (1 - 2o), \]

\[
X_3 = 8o - 12o^2 - 4 - 9b^4 - 2 (o + 4o^2 + 12) b^3 + (8o - 29o^2 - 28)b^2
\]
\[-16 (1 - o + 2o^2) b + c^2 (b + 1)^3 (2o - 1)^2, \]
Proof: We solve Game TI by backward induction. We start in stage 2 by solving the retailer’s and the manufacturer’s problems simultaneously given by

\[
\max_{P_f} R = (p_f - w) \left[ (1 - o)a - p_f + b (p_{m0} - p_f) + ci \right],
\]
\[
\max_{P_{m0}} M = w \left[ (1 - o)a - p_f + ci \right] + p_{m0} \left[ (oa - p_{m0} + b (p_f - p_{m0}) + ci \right] - i^2,
\]

and get the price reaction functions \( p_{RF(TI)}^{P_f} \) and \( p_{RF(TI)}^{P_{m0}} \) by solving the system of first-order optimality conditions \( \frac{\partial R}{\partial P_f} = \frac{\partial M}{\partial P_{m0}} = 0 \). The retailer’s and manufacturer’s concavity conditions are negative and given by \((-2 - 2b)\), and are therefore verified for all parameters \( o, c, b \in (0, 1) \). These reaction functions are given by

\[
p_{RF(TI)}^{P_f} = \frac{(3b^2 + 4b + 2) w + 2 (1 - o) + b(2 - o) + (3b + 2) ci}{(b + 2)(3b + 2)},
\]
\[
p_{RF(TI)}^{P_{m0}} = \frac{3b (b + 1) w + b + bo + 2o + (3b + 2) ci}{(b + 2)(3b + 2)}.
\]

The prices \( p_{m0} \) and \( p_f \) are then replaced by the obtained reaction functions \( (p_{RF(TI)}^{P_f} \) and \( p_{RF(TI)}^{P_{m0}} \)) in the manufacturer’s problem in stage 1, which is given by

\[
\max_{w,i} M = w \left[ (1 - o)a - p_{RF(TI)}^{P_f} + ci \right] - i^2
\frac{\partial M}{\partial w}, \quad \frac{\partial M}{\partial i} = 0. \]

We get the equilibrium wholesale price and service level by solving her first-order optimality conditions \( \frac{\partial M}{\partial w} = \frac{\partial M}{\partial i} = 0 \). The manufacturer’s second order conditions
are given by

\[ \frac{c^2(1 + b) - 2(b + 2)^2}{(b + 2)^2} < 0, \]  

(15)

\[ \frac{N}{(b + 2)^2 (3b + 2)} > 0, \]  

(16)

with \( N = -(2b + 1)(-96b + 40bc^2 + 45b^2c^2 + 18b^3c^2 - 100b^2 - 36b^3 + 12c^2 - 32). \)

Note that for any \( o, c, b \in (0, 1) \), both conditions in (15) and (16) are always satisfied. Therefore, the concavity conditions for the manufacturer’s problem in Game T are verified for all parameters \( o, b, c \in (0, 1) \). The obtained solutions in (9) and (10) are then inserted into the reaction functions \( p_f^{RF(TI)} \) and \( p_m^{RF(TI)} \) to get the equilibrium prices as function of the model parameters.

Note that the equilibrium solution satisfies the conditions that \( i^{TI}, (p_f^{TI} - w^{TI}), p_m^{TI} \) and \( R^{TI} \) are always positive for any \( o, c, b \in (0, 1) \). Further, the denominator of \( i^{TI} \) is always negative, and, from the reaction functions, for any \( w^{TI} > 0 \), \( p_f^{TI} \) is always positive.

Lastly, the condition that guarantees positive wholesale price \( (w^{TI}) \), offline demand \( (q_f^{TI}) \), online demand \( (q_m^{TI}) \), manufacturer’s profit \( (M^{TI}) \), and a manufacturer’s online price that is higher than its wholesale price is as follows:

\[ o > \max(J_1, J_3), \]
\[ o < \min(J_2, J_5), \]
\[ J_4 < 0, \]
where

\[
J_1 = \frac{-(6 - 6c^2) b^3 + (36 - 19c^2) b^2 + (44 - 19c^2) b + (16 - 6c^2)}{2((6c^2 - 12) b^3 + (19c^2 - 43) b^2 + (19c^2 - 46) b + (6c^2 - 16))},
\]

\[
J_2 = \frac{4 - c^2}{4 - 2c^2},
\]

\[
J_3 = \frac{(4b + 7bc^2 + 8b^2c^2 + 3b^3c^2 + 8b^2 + 6b^3 + 2c^2)}{((3c^2 - 6) b^3 + (8c^2 - 21) b^2 + (7c^2 - 22) b + (2c^2 - 8))},
\]

\[
J_4 = \frac{(4b^3c^2 + 12b^2c^2 + 12bc^2 + 4c^2 - 32b - 12 - 29b^2 - 8b^3) o^2}{-2(-8b + 6bc^2 + 6b^2c^2 + 2b^3c^2 - 4b^2 + b^3 + 2c^2 - 4) o + ((c^2 - 24) b^3 - 9b^4 + (3c^2 - 28) b^2 + (3c^2 - 16) b + (c^2 - 4))},
\]

\[
J_5 = \frac{-(6b^3c^2 - 18b^4 + 16b^2c^2 - 64b^3 + 14bc^2 - 96b^2 + 4c^2 - 64b - 16)}{(-12b^3c^2 + 14b^3 - 32b^2c^2 + 48b^2 - 28bc^2 - 8c^2 + 48b + 16)}.
\]

The expression \( J_1 \) is strictly decreasing in both \( b \) \((\frac{\partial J_1}{\partial b} < 0)\) and \( c \) \((\frac{\partial J_1}{\partial c} < 0)\). Therefore, \( \min(J_1(b, c)) = J_1(0, 0) = 0.388 \) and \( \max(J_1(b, c)) = J_1(1, 1) = 0.5 \).

Hence, \( o > J_1 \) is a valid condition for this game. Similarly, \( \frac{\partial J_2}{\partial c} > 0 \) for any \( c \in (0, 1) \). Therefore, the \( \min(J_2(c)) = J_2(0) = 1.5 \). Hence, the condition \( o < J_2 \) is always verified for any \( o \in (0, 1) \). Further, for \( o, b, c \in (0, 1) \), \( J_3 < 0 \), therefore the condition \( o > J_3 \) is always verified.

\( J_4 \) is a second degree polynomial in \( o \). The discriminant of this polynomial is given by \( \Delta \) such as:

\[
\Delta = (2b + 1)(b + 1)^3 [(18c^2 - 36) b^3 + (45c^2 - 100) b^2
+ (40c^2 - 96) b + (12c^2 - 32)].
\]
For any given \( o, b, c \in (0, 1) \), \( \Delta < 0 \). Therefore, there are no real roots to \( J_4 \) in \( o \) and \( J_4 \) has the same sign for any \( o, \in (0, 1) \). We can then determine the sign of \( J_4 \) by setting \( o = 0 \) and computing \( J_4 \) value, which is then given by:

\[
(c^2 - 24) b^3 - 9b^4 + (3c^2 - 28) b^2 + (3c^2 - 16) b + (c^2 - 4).
\]

Since this expression is negative for any \( b, c \in (0, 1) \), then \( J_4 < 0 \) for any given \( o, b, c \in (0, 1) \). Hence, the condition \( o > J_4 \) is always verified. Finally, the expression \( J_5 \) is strictly increasing in both \( b \left( \frac{\partial J_5}{\partial b} > 0 \right) \) and \( c \left( \frac{\partial J_5}{\partial c} > 0 \right) \). Therefore, \( \min(J_5 (b, c)) = J_5 (0, 0) = 1 \). Since \( o < 1 \) by model set-up, then the condition \( o < J_5 \) is always verified.

Therefore, the necessary and sufficient condition for an interior equilibrium solution in Game TI is

\[
o > J_1.
\]

This condition defines the feasibility condition for Game TI. It is represented in Figure 1.2.

### 6.2 Comparisons between games

To compare the equilibrium solutions and outputs between games, we write the analytical expressions of each comparison and seek the sign of each expression analytically, considering the feasible domain for each Game. When we cannot determine these signs analytically, we plot these expressions as implicit plots for \( b \in (0, 1) \) and for acceptable values of \( o \) considering the feasibility condition for the games being compared. These plots are obtained for \( c \in (0, 1) \) at an increment of 0.04.
6.2.1 Comparisons of Games T and I

Comparisons of profits

Comparisons of equilibrium profits obtained in Games TI and I are reported in the second column of Table 2 \((T - I)\). The analytical expressions of these comparisons are as follows:

\[
M^T - M^I = \frac{X_5}{4(2b + 1)(b + 1)(16b + 9b^2 + 8)(c^2 - 8)},
\]
\[
R^T - R^I = \frac{X_6}{(1 + b)(9b^2 + 16b + 8)^2(8 - c^2)^2},
\]

where

\[
X_5 = (8 - c^2) [9b^4 + 2(4o^2 + o + 12)b^3 + (29o^2 - 8o + 28)b^2 + 16(2o^2 - o + 1)b
+ 4(1 - 2o + 3o^2)] - 4(2b + 1)(b + 1)(16b + 9b^2 + 8),
\]
\[
X_6 = \left( 9 \left( c^2 - 8 \right)^2 (o - 1)^2 - 1476 \right) b^4 - 324b^5 + \left( 24 \left( c^2 - 8 \right)^2 (o - 1)^2 - 2752 \right) b^3
+ \left( 28 \left( c^2 - 8 \right)^2 (o - 1)^2 + 2624 \right) b^2 + \left( 16 \left( c^2 - 8 \right)^2 (o - 1)^2 + 1280 \right) b
+ \left( 4 \left( c^2 - 8 \right)^2 (o - 1)^2 - 256 \right). \]

It’s easy to see that \(X_6\) is always negative for any \(c, o, b \in (0, 1)\). Therefore, \(R^T < R^I\). We cannot determine the sign of \(X_5\) and of the total profit comparisons expression analytically, so we plot these expressions considering the feasibility condition for Game T in \((8)\), since the Game I equilibrium solution is feasible for all \(c \in (0, 1)\), in order to find the results reported in Table 2.

Insert Figures A.1 and A.2
Comparisons of prices and demands

\[
\begin{align*}
 w^T - w^I &= \frac{1}{2} \frac{Y_1}{(c^2 - 8)(1 + 2 b)(1 + b)(9 b^2 + 16 b + 8)}, \\
p_f^T - p_f^I &= \frac{1}{2} \frac{Y_2}{(c^2 - 8)(1 + 2 b)(1 + b)(9 b^2 + 16 b + 8)}, \\
q_f^T - q_f^I &= \frac{Y_3}{(c^2 - 8)(9 b^2 + 16 b + 8)},
\end{align*}
\]

where

\[
\begin{align*}
Y_1 &= (9 c^2 + 72) b^4 + 8 (1 - o) c^2 + 64 o + ((-7 o + 32) c^2 + 56 o + 216) b^3 \\
&\quad + ((-24 o + 48) c^2 + 192 o + 200) b^2 + ((-24 o + 32) c^2 + 192 o + 64) b, \\
Y_2 &= (9 c^2 + 144) b^4 + 12 (1 - o) c^2 + 96 o + [(-19 o + 44) c^2 + 152 o + 356] b^3 \\
&\quad + ((-46 o + 70) c^2 + 368 o + 316) b^2 + ((-40 o + 48) c^2 + 320 o + 96) b, \\
Y_3 &= (16 - b^2 (3c^2 - 24) - 2c^2 - b (4c^2 - 42)) o + (4bc^2 + b^2 (3c^2 - 6) + 2c^2).
\end{align*}
\]

It is easy to see that the expressions \((w^T - w^I)\) and \((p_f^T - p_f^I)\) are negative for any values of \(o, c, b \in (0, 1)\). Finally, \((q_f^T - q_f^I)\) is positive iff \(Y_3 < 0\), which is equivalent to \(o < Y_4(b, c)\), such that \(Y_4(b, c) = \frac{-4bc^2 + b^2 (3c^2 - 6) + 2c^2}{16 - b^2 (3c^2 - 24) - 2c^2 - b (4c^2 - 42)}\). The expression \(Y_4\) is strictly increasing in \(b\) \((\frac{\partial Y_4}{\partial b} > 0)\) and decreasing in \(c\) \((\frac{\partial Y_4}{\partial c} < 0)\). Therefore, \(\max(Y_4) = Y_4(1, 0) = 0.073\). Given that \(o > 0.43\) in all feasible domains of Game T, then \(o > Y_4\) and \(q_f^T < q_f^I\).
6.2.2 Comparisons between Games TI and T

Comparisons of profits

Comparisons of equilibrium profits obtained in Games TI and T are reported in the third column of Table 5 (TI – T). The analytical expressions of these comparisons are as follows.

\[
M^{TI} - M^T = \frac{-c^2 (9b^3 + 5b^2 o + 20b^2 + 8bo + 16b + 4o + 4)^2}{4 (1 + b) (9b^2 + 16b + 8) (18b^3 c^2 + 45b^2 c^2 - 36b^3 + 40bc^2 - 100b^2 + 12c^2 - 96b - 32)^2} > 0.
\]

Therefore \( M^{TI} - M^T > 0 \), for any \( b, c \) and \( o \) given the feasibility condition for Game T in (8) and for Game TI in (17).

\[
R^{TI} - R^T = \frac{(3b^2 + 4b + 2)^2 c^2 (9b^3 + 5b^2 o + 20b^2 + 8bo + 16b + 4o + 4) \Omega}{(18b^3 c^2 + 45b^2 c^2 - 36b^3 + 40bc^2 - 100b^2 + 12c^2 - 96b - 32)^2 (9b^2 + 16b + 8)^2 (1 + b)^2},
\]

with

\[
\Omega = \left( b \left( 88c^2 - 192 \right) + b^3 \left( 36c^2 - 72 \right) + b^2 \left( 95c^2 - 200 \right) + 28c^2 - 64 \right) o \\
+ \left( 64 - b^3 \left( 27c^2 - 72 \right) - b^2 \left( 70c^2 - 200 \right) - 20c^2 - b \left( 64c^2 - 192 \right) \right).
\]

Since the denominator of \( (R^{TI} - R^T) \) is always positive, we focus on the study of the numerator. The latter is positive if and only if \( \Omega > 0 \), which is equivalent to \( o < Y_5 (b, c) \), such that \( Y_5 (b, c) = \frac{-192b + 64b^2 c^2 + 70b^2 c^2 + 27b^3 c^2 - 200b c^2 - 192b^3 + 200b c^2 - 200c^2 + 28c^2 + 64}{-192b + 886c^2 - 192b c^2 + 36b^2 c^2 - 200b c^2 - 72b^3 + 28c^2 - 64} \). The expression \( Y_5 \) is strictly increasing in both \( b \) (\( \frac{\partial Y_5}{\partial b} > 0 \)) and \( c \) (\( \frac{\partial Y_5}{\partial c} > 0 \)). Therefore, \( \min(Y_5) = Y_4 (0, 0) = 1 \). Given that \( o < 1 \), then \( o < Y_5 \) in all feasible domains, which means that \( \Omega > 0 \) and \( R^{TI} > R^T \) in all feasible domains.

Since \( M^{TI} - M^T > 0 \) and \( R^{TI} - R^T > 0 \), then the sum of these expressions is
Comparisons of prices and demands  Comparisons of equilibrium prices and demands obtained in Games TI and I are reported in the third column of Table 2 (TI – T). The analytical expressions of these comparisons are as follows:

\[
\begin{align*}
 w_{TI} - w_{T} &= \frac{-c^2(3b+2)(3b^2+6b+4)(9b^3+5b^2o+20b^2+80o+16b+40+4)}{2(1+b)(9b^2+16b+8)(18b^3+c^2-36b^2+40bc^2-100b^2+12c^2-96b-32)}, \\
 p_{f TI} - p_{f T} &= \frac{-c^2(9b^3+30b^2+32b+12)(9b^3+5b^2o+20b^2+80o+16b+40+4)}{2(1+b)(9b^2+16b+8)(18b^3+c^2-36b^2+40bc^2-100b^2+12c^2-96b-32)}, \\
 p_{mo TI} - p_{mo T} &= \frac{-c^2(3b+4)(3b+2)(9b^3+5b^2o+20b^2+80o+16b+40+4)}{2(9b^2+16b+8)(18b^3+c^2-36b^2+40bc^2-100b^2+12c^2-96b-32)}, \\
 q_{f TI} - q_{f T} &= \frac{-c^2(9b^3+5b^2o+20b^2+80o+16b+40+4)(c^2(3b^2+4b+2))}{(9b^2+16b+8)(18b^3+c^2-36b^2+40bc^2-100b^2+12c^2-96b-32)}, \\
 q_{mo TI} - q_{mo T} &= \frac{-c^2(3b+2)(4b^2+7b+4)(9b^3+5b^2o+20b^2+80o+16b+40+4)c^2}{2(1+b)(9b^2+16b+8)(18b^3+c^2-36b^2+40bc^2-100b^2+12c^2-96b-32)}. 
\end{align*}
\]

It is easy to see that all these expressions are positive for any \(c, b, o \in (0, 1)\).

6.2.3 Comparisons between Games TI and I

Comparisons of profits  Comparisons of equilibrium profits obtained in Games TI and I are reported in the fourth column of Table 2 (TI – I). The analytical expressions of these comparisons are as follows:

\[
\begin{align*}
 M_{TI} - M_{T} &= \frac{(8-c^2)\times 3-(1+2b)(18b^3+c^2+45b^2c^2-36b^2+40bc^2-100b^2+12c^2-96b-32)}{(1+2b)(18b^3+c^2+45b^2c^2-36b^2+40bc^2-100b^2+12c^2-96b-32)(8-c^2)}, \\
 R_{TI} - R_{T} &= \frac{(2c^2o-c^2-4o+4)c^2(3b^2+4b+2)(8-c^2)^2-4(18b^3+c^2+45b^2c^2-36b^2+40bc^2-100b^2+12c^2-96b-32)^2}{(18b^3+c^2+45b^2c^2-36b^2+40bc^2-100b^2+12c^2-96b-32)(8-c^2)^2}. 
\end{align*}
\]

We cannot determine the signs of these expressions and of the total profit comparisons analytically, so we plot these expressions for \(b \in (0, 1)\) and for acceptable values of \(o\) considering the feasibility condition for Game TI in (17). These plots are obtained for \(c \in (0, 1)\), at an increment of 0.04, since the Game I equilibrium solution is feasible for all \(c \in (0, 1)\). We find the results reported in Table 2.
Comparisons of prices and demands  Comparison of equilibrium prices and demand in games TI and I gives the following results:

\[ w^{TI} - w^I = \frac{Z_1}{(1+2b)(18b^3c^2+45b^2c-36b^2+4b^2c^2-100b^2+12c^2-96b-32)(c^2-8)}, \]

\[ p_f^{TI} - p_f^I = \frac{Z_2}{(1+2b)(18b^3c^2+45b^2c-36b^2+4b^2c^2-100b^2+12c^2-96b-32)(c^2-8)}, \]

\[ q_f^{TI} - q_f^I = \frac{Z_3}{(18b^3c^2+45b^2c^2-36b^2+4b^2c^2-100b^2+12c^2-96b-32)(c^2-8)}, \]

where

\[ Z_1 = -2 (c^2 - 8) \left(-24b + 14bc^2 + 16b^2c^2 + 6b^2c^2 - 24b^2 - 7b^3 + 4c^2 - 8\right) o+2(-64b+40bc^2 + 7bc^4 + 138b^2c^2 + 160b^3c^2 + 8b^2c^4 + 63b^4c^3 + 3b^3c^4 - 200b^2 - 216b^3 - 72b^4 + 2c^4), \]

\[ Z_2 = -2 (c^2 - 8) \left(-40b + 22bc^2 + 27b^2c^2 + 12b^3c^2 - 46b^2 - 19b^3 + 6c^2 - 12\right) o + b (22c^4 + 112c^2 - 192) + b^2 (27c^4 + 394c^2 - 632) + b^3 (12c^4 + 464c^2 - 712) + b^4 (198c^2 - 288) + 6c^4, \]

\[ Z_3 = -2 (c^2 - 2) (c^2 - 8) \left(b + 1\right) (4b + 3b^2 + 2) o + b (6c^2 + 8c^2) + b^2 (7c^4 + 6c^2 + 24) + b^3 (3c^4 + 24) + 2c^4. \]

Since the denominators of all these expressions are always positive, we focus on studying the numerators.

\[ Z_1 > 0 \iff o < Y_6 = \frac{(63c^2-72)b^6 + (3c^4+160c^2-216)b^3 + (8c^4+138c^2-200)b^2 + (7c^4+40c^2-64)b + 2c^4}{(c^2-8)(-24b+14bc^2+16b^2c^2+6b^2c^2-24b^2-7b^3+4c^2-8)}, \]

\[ Z_2 > 0 \iff o < Y_7 = \frac{b(22c^4+112c^2-192)+b^2(27c^4+394c^2-632)+b^3(12c^4+464c^2-712)+b^4(198c^2-288)+6c^4}{2(c^2-8)(-40b+22bc^2+27b^2c^2+12b^3c^2-46b^2-19b^3+6c^2-12)}, \]

\[ Z_3 > 0 \iff o < Y_8 = \frac{b(6c^2+8c^2)+b^2(7c^4+6c^2+24)+b^3(3c^4+24)+2c^4}{2(c^2-2)(c^2-8)(b+1)(4b+3b^2+2)}. \]

The expression \( Y_6 \) is strictly increasing in both \( b \left( \frac{\partial Y_6}{\partial b} > 0 \right) \) and \( c \left( \frac{\partial Y_6}{\partial c} > 0 \right) \).

Therefore, \( \min(Y_8) = Y_8(0,0) = 0 \) and \( \max(Y_8) = Y_8(1,1) = 0.317 \). Given the feasibility condition for Game TI in (17), \( o > Y_8 \) in all feasible domains, which means that \( Z_3 < 0 \) and \( q_f^{TI} < q_f^I \) in all feasible domains.
We cannot determine the signs of the expressions $Y_6$ and $Y_7$ and of their derivatives with respect to $b$ and $c$ analytically. Therefore, we plot $Y_6$ and $Y_7$ considering the feasibility condition for Game TI in (17). The Game I equilibrium solution is feasible for all $c \in (0, 1)$, in order to find the results reported in Table 2.

Table 1: Models

<table>
<thead>
<tr>
<th></th>
<th>Game $T$</th>
<th>Game $I$</th>
<th>Game TI</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Demand functions</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Online demand ($q_{mo}$)</td>
<td>$oa - p_{mo}$</td>
<td>0</td>
<td>$oa - p_{mo} + ci$</td>
</tr>
<tr>
<td></td>
<td>$+b(p_f - p_{mo})$</td>
<td>+$b(p_f - p_{mo})$</td>
<td></td>
</tr>
<tr>
<td>Offline demand ($q_f$)</td>
<td>$(1 - o)a - p_f$</td>
<td>$a - p_f + ci$</td>
<td>$(1 - o)a - p_f + ci$</td>
</tr>
<tr>
<td></td>
<td>$+b(p_{mo} - p_f)$</td>
<td>+$b(p_{mo} - p_f)$</td>
<td></td>
</tr>
<tr>
<td><strong>Profit functions</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Manufacturer ($M$)</td>
<td>$wq_f + p_{mo}q_{mo}$</td>
<td>$wq_f - i^2$</td>
<td>$wq_f + p_{mo}q_{mo} - i^2$</td>
</tr>
<tr>
<td>Retailer ($R$)</td>
<td>$(p_f - w)q_f$</td>
<td>$(p_f - w)q_f$</td>
<td>$(p_f - w)q_f$</td>
</tr>
</tbody>
</table>
Table 2: Summary of comparisons across games

<table>
<thead>
<tr>
<th></th>
<th>$(T - I)$</th>
<th>$(TI - T)$</th>
<th>$(TI - I)$</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Effects on profits</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Retailer profit</td>
<td>$-$</td>
<td>$+$</td>
<td>$-$</td>
</tr>
<tr>
<td>Manufacturer profit</td>
<td>$\pm$</td>
<td>$+$</td>
<td>$\pm$</td>
</tr>
<tr>
<td>Total channel profit</td>
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<td>$\pm$</td>
</tr>
<tr>
<td><strong>Effects on prices</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Retail price offline</td>
<td>$-$</td>
<td>$+$</td>
<td>$-$</td>
</tr>
<tr>
<td>Manufacturer price online</td>
<td></td>
<td>$+$</td>
<td></td>
</tr>
<tr>
<td>Wholesale price</td>
<td>$-$</td>
<td>$+$</td>
<td>$-$</td>
</tr>
<tr>
<td><strong>Effects on demand</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Retailer demand offline</td>
<td>$-$</td>
<td>$+$</td>
<td>$-$</td>
</tr>
<tr>
<td>Manufacturer demand online</td>
<td></td>
<td></td>
<td>$+$</td>
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</tbody>
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