Cooperative advertising programs: Are accrual constraints necessary?

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Abstract

This paper investigates how the use of an accrual constraint in a cooperative advertising program affects channel members’ profits in a bilateral monopoly, as well as their pricing and advertising decisions. The main findings indicate that, compared to unconstrained cooperative advertising programs, when an accrual constraint is used and the manufacturer’s contribution to the retailer’s advertising costs exceeds the accrued cooperative advertising budget, the retailer reduces both her retail price and advertising efforts to the level where cooperative advertising is not offered; while the manufacturer also reduces his wholesale price and advertising efforts, but this time, the wholesale price remains higher than when there is no cooperative advertising. These strategic moves translate to less (more) profits for the manufacturer (retailer). The use of an accrual constraint is counterproductive for the manufacturer as the retailer uses the accrued advertising fund as a side payment rather than a direct incentive to invest more in advertising. The manufacturer and retailer are better off when unconstrained cooperative advertising programs are supplemented with other incentives, including side payments and advertising support services.

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1 Introduction

Cooperative advertising programs between manufacturers and retailers are financial arrangements in which manufacturers commit to pay part or full costs of advertising undertaken by retailers to promote locally the manufacturers' products. These programs are generally developed for various reasons, including providing additional incentives to retailers to promote the manufacturers' products, stimulating immediate sales, and increasing the effectiveness of manufacturers' promotional activities, which may include advertising programs at the national or regional levels. Recent reports indicate that American businesses offered about $36 billion in cooperative advertising money to their vendors in 2015, which represents 12% of their total advertising costs (Borrell Associates Report 2015).

As observed by Dutta et al. (1995), cooperative advertising arrangements generally contain two major components: a participation rate that specifies the percentage of retail advertising that a manufacturer is committed to pay and an accrual rate that specifies the maximum manufacturer's contribution to the cooperative advertising program as a percentage of the retailer's purchases from the manufacturer. Both the accrual and participation rates are equally important in a cooperative advertising arrangement. The first is linked to purchases and communicates the maximum amount a manufacturer allocates to a retailer's local advertising expenses. The second indicates how the costs between the two partners will be shared. The marketing firm MultiAd, described as "the largest co-op database ever accumulated", indicates both accrual and participation rates as important elements of the cooperative advertising agreement (MultiAd website 2015).

There is an extensive literature in supply chain and marketing channels that investigates the optimal design of cooperative advertising arrangements in various channel structures and contexts. Two comprehensive reviews of this literature have recently been conducted by Aust and Buscher (2014) and Jørgensen and Zaccour
These reviews summarize the key findings on the design and use of participation rates in cooperative advertising programs and their various impacts. The main results from this literature suggest that, in the context of a bilateral monopoly, these programs are effective in boosting retail advertising, expanding demand, and ultimately increasing profits for each channel member (e.g., Dant and Berger 1996; Jørgensen et al. 2000; Huang and Li 2001; Yue et al. 2006; Karray and Zaccour 2006; Xie and Ai 2006; Yan 2010; Kunter 2012; Yang et al. 2013; Karray 2013; Zhang et al. 2013).

A few studies have expanded these results to the case where some competition arises in the channel. In particular, Bergen and John (1997) used a consumer-based model and showed that cooperative advertising programs can benefit a single manufacturer selling through multiple retailers. Karray and Zaccour (2007) found that cooperative advertising can lead to a prisoner’s dilemma situation for manufacturers when competition arises at both levels of the channel. Other researchers have recently showed that these programs are not always effective for competing retailers (Chutani and Sethi 2012; Karray and Amin 2015; Liu et al. 2014; Karray 2015). In particular, cooperative advertising may not benefit competing retailers because it can increase prices, thereby cancelling out its demand-stimulating effects.

These developments in the literature show the following three important points. First, it is important to endogenize prices in cooperative advertising models. Second, competitive interactions in the channel significantly affect cooperative advertising effectiveness. Third, despite the large number of analytical studies about cooperative advertising, the literature has overlooked the fact that a very large number of such programs offered by manufacturers specify an accrual constraint in addition to a participation rate.

Explicitly or implicitly, the existing literature about cooperative advertising assumes that manufacturers offer unlimited cooperative advertising support to their retailers, i.e., no matter how much the retailer’s advertising expenses amount to, the manufacturer commits to sharing all of these expenses according to the agreed participation rate. Alternatively, the dismissal of accrual rates in the existing literature could mean that even when manufacturers set a budget to limit their cooperative advertising support to retailers, the retailer’s advertising expenditures do exceed the maximum amount that the manufacturer is willing to reimburse. While such unconstrained cooperative advertising programs exist in the marketplace, there are many others in which manufacturers use accrual rates as a way to control their budget allocations to cooperative advertising activities. This is shown in the empirical study by Dutta et al. (1995) who found that 2,156 cooperative advertising programs in various industries offer both the participation and accrual rates. Numerous more
recent examples can be found in the Co-op Advertising Programs Sourcebook, a compiled database of cooperative advertising programs offered by manufacturers in different industries. For instance, in the sporting goods industry, the manufacturer Yonex Corporation offers its retailers a 50 percent participation rate and a 5 percent accrual rate, while Nike offers 100% participation rate and only 0.5 percent accrual rate. This shows that many manufacturers include information about the accrual rate along with the cooperative advertising rates in their communications to the retailers.

To the best of our knowledge, the paper by Zhang et al. (2015) is the first attempt that formally investigates the use of an accrual rate in a bilateral monopoly supply chain. In a dynamic setup in which pricing decisions are exogenous, the authors consider that the manufacturer sets national advertising effort and a cooperative advertising participation rate and the retailer determines local advertising effort, taking the accrual rate as given. The main results indicate that some of the previous findings in the cooperative advertising literature may not generalize to programs that have accrual constraints. As an example, while it is known that without an accrual constraint retailers always increase their advertising efforts as the participation rate increases, Zhang et al. (2015) find that when an accrual constraint is added, the participation rate could have the opposite effect on the retailer’s advertising. They also find that the accrual constraint reduces both the manufacturer’s and retailer’s advertising levels and impacts negatively on the two channel partners’ profits. Such a dramatic change on what we know about some elements of cooperative advertising programs calls for additional research on the impact of the accrual constraint on channel members’ strategies and profits.

Contrary to Zhang et al.’s work, in addition to their advertising decisions, the current research allows both the manufacturer and retailer to endogenously set their respective prices. The goal of this research is therefore to investigate the impact of the use of an accrual constraint on channel members’ profits and pricing and advertising decisions. We hope our research will provide useful practical guidelines to marketing managers and contribute to expanding our current limited knowledge about the use of accrual rates in cooperative advertising programs.

The remainder of the paper is organized as follows. Section 2 describes the model and discusses its assumptions. Section 3 derives the equilibrium solutions for three games, namely the game without cooperative advertising and the cooperative advertising games with and without an accrual constraint. Section 4 assesses the impact of the accrual constraint on channel members’ optimal strategies and profits. Section 5 concludes and discusses the managerial and theoretical implications of this

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2 The model

Consider a bilateral monopoly in which a manufacturer sells its product to a single retailer who then sells this product to consumers. Let $w$ and $p$ denote, respectively, the wholesale price set by the manufacturer and the retail price set by the retailer. Also, let $e_m$ and $e_r$ denote the manufacturer’s and retailer’s advertising efforts for the product. In addition to setting his pricing and advertising decisions, the manufacturer also commits to stimulate retail advertising. The goal is to give additional incentives to the retailer to advertise more and secure better profits. As a consequence, in addition to advertising for his product, the manufacturer offers to share part of the retailer’s advertising expenses at a given rate, $\eta$ (coop participation rate), to a maximum of percentage, $r$ (accrual rate), of the retailer’s purchases.

We assume the following linear demand function:

$$d = A + b e_m + g e_r - p$$

The parameter $A$ is positive and represents the baseline demand for the product. The parameters $b$ and $g$ respectively denote the effects of manufacturer and retailer advertising. This work relies on several assumptions. We discuss some of them in the following lines. The parameter $A$ serves as a scale and its value does not significantly change the qualitative results in this paper. The effects of retail price on the demand is normalized to 1. Because retail prices are generally known to have greater effects on demand than advertising, we assume that the effects of both manufacturer and retailer advertising ($b$ and $g$) lie between 0 and 1. We consider that the accrual rate ($r$) is exogenous to the model to focus on its impacts on channel members’ strategies and profits. While this assumption may seem restrictive, it can be justified by the observation that manufacturers often set their accrual rates and keep them unchanged for very long periods that can go over several years (see examples in the Co-op Advertising Programs Sourcebook). Also, the accrual rate is more influenced by financial than marketing considerations. It is generally set to limit advertising expenditures rather than to pursue any identified market opportunity. On the other hand, manufacturers can adjust their pricing and other advertising decisions on a regular basis. Retailers also have the possibility to react to any changes in the manufacturers’ pricing and advertising decisions.

We assume that the retailer does not stockpile, i.e., the quantity of product purchased from the manufacturer is identical to the quantity sold to consumers.
This assumption ensures that the manufacturer’s cooperative advertising program aligns the interests of the manufacturer with those of the retailer. Everything else being equal, the manufacturer sets a cooperative advertising budget that depends on the sales of his product.

Advertising costs for the manufacturer and retailer are convex and given respectively by $C(e_m) = e_m^2$ and $C(e_r) = e_r^2$. Convex advertising costs are common in the marketing literature (Aust and Buscher 2014). They imply that marginal costs of advertising are increasing. When the manufacturer offers an unconstrained cooperative advertising program with a participation rate $\eta$ to the retailer ($\eta \in (0,1)$), the manufacturer’s portion of the retailer’s advertising expenses on her product is $\eta e_r^2$, while the retailer’s effective advertising cost is $(1 - \eta)e_r^2$. Alternatively, when the manufacturer offers a constrained cooperative advertising program with a participation rate $\eta$, and an accrual rate $r$, the manufacturer’s constrained contribution to the retailer’s advertising expenses is denoted by $CR$, which is given by:

$$CR = \min\left(rwd, \eta e_r^2\right).$$

In this case, the manufacturer pays a percentage of the retailer’s advertising expenses $\eta e_r^2$ as in the unconstrained cooperative advertising program to a maximum amount of $rwd$, which is the accumulated accrued advertising budget, also known as available cooperative advertising funds. In any case, the manufacturer and retailer determine their respective advertising and pricing decisions so as to maximize their own profits. The manufacturer’s profit function, $\Pi_m$, and the retailer’s profit, $\Pi_r$, are respectively given by:

$$\max_{w, e_m, \eta} \Pi_m = wd - e_m^2 - CR \quad \text{and} \quad \max_{p, e_r} \Pi_r = (p - w)d - e_r^2 + CR,$$

where $CR = \min\left(rwd, \eta e_r^2\right)$.

## 3 Equilibrium solutions

We use the Stackelberg solution concept to derive equilibria. The manufacturer is the channel leader, while the retailer is the follower. We first analyze the status quo scenario in which the manufacturer offers no cooperative advertising to the retailer. We follow with the analysis of the second scenario in which the manufacturer offers an unconstrained cooperative advertising program. This is the most studied scenario in the literature: there is no accrual constraint and the manufacturer reimburses the retailer for advertising spending as per the cooperative advertising rate. We finish with
the analysis of the scenario where the manufacturer offers a cooperative advertising support subject to an accrual constraint. In each scenario, the manufacturer first announces his wholesale price, participation rate, and advertising effort. Knowing these announcements, the retailer sets his retail price and advertising effort.

To obtain subgame-perfect equilibria, we derive the optimal solutions backwards. The retailer’s problem is solved first and the resolution of the manufacturer’s problem comes last. Recall that the manufacturer’s accrual rate \( r \) is exogenous and is set before the start of the game. It is therefore treated as a parameter in our model rather than a decision variable.

We define the accrual constraint for the manufacturer (\( AC \)) as:

\[
AC = rwd - \eta e^2.
\]

If the accrual constraint is not activated or satisfied (i.e., \( AC \geq 0 \)), the retailer’s share of the advertising costs for the manufacturer’s product is lower than or equal to the accrued cooperative advertising funds. In this case, the games with and without accrual constraint give the same solution at the equilibrium. Conversely, if the accrual constraint is activated or not satisfied (i.e., \( AC < 0 \)), the retailer’s share of the advertising costs exceeds the accrued cooperative advertising funds, and the two games lead to different strategies at equilibrium.

For each problem, we need to verify the following conditions at the equilibrium: retailer’s pricing, advertising effort, and profits are positive and the retail price is higher than the wholesale price to guarantee positive retail margins. We also verify that the manufacturer’s wholesale price, participation rate, and advertising effort are positive. Finally, demands are positive and second-order conditions for all problems are verified. The proof for these conditions is included in Appendices A, B, and C.

4 No cooperative advertising

In this case, the manufacturer does not offer a cooperative advertising program to the retailer, i.e., \( CR = \eta = 0 \). The equilibrium pricing and advertising decisions are denoted by the superscript \( NCA \) (No Cooperative Advertising) and are summarized in the following proposition.

**Proposition 1** For the case where cooperative advertising is not offered in the channel, the equilibrium pricing and advertising decisions and profits for the manufacturer
and retailer are given by:

\[
\begin{align*}
\theta^NCA &= \frac{A(g^2 - 4)}{b^2 + 2g^2 - 8}, \\
\Phi^NCA &= \frac{3A(g^2 - 6)}{b^2 + 2g^2 - 8}, \\
\epsilon^NCA_m &= -\frac{Ab}{b^2 + 2g^2 - 8}, \\
\epsilon^NCA_r &= -\frac{Ag}{b^2 + 2g^2 - 8}, \\
\Pi^NCA_m &= -\frac{A^2}{b^2 + 2g^2 - 8}, \\
\Pi^NCA_r &= \frac{A^2(4 - g^2)}{(b^2 + 2g^2 - 8)^2}.
\end{align*}
\]

**Proof.** See Appendix A. 

Given the model’s assumptions and parameters’ ranges, the player’s strategies and profits in Proposition 1 are positive. Therefore there is no additional restriction to impose on the model parameters to ensure positive strategies and non-negative profits.

### 4.1 Cooperative advertising without accrual constraint

The derivation of the players’ optimal strategies when the manufacturer offers an unconstrained cooperative advertising program is described in Appendix B. The equilibrium pricing and advertising decisions are denoted by the superscript \( \text{NC} \) (No Constraint) and are summarized in Proposition 2 below.

**Proposition 2** For the case where cooperative advertising is offered without an accrual constraint or when the accrual constraint is not active (i.e., \( rwd - \eta \epsilon_r^2 \geq 0 \)),
the equilibrium strategies and profits for the two channel members are as follows:

\[
\begin{align*}
  w_{NC} &= \frac{A(3g^2 - 16)}{4b^2 + 9g^2 - 32}, \\
p_{NC} &= \frac{3A(g^2 - 8)}{4b^2 + 9g^2 - 32}, \\
ed_{NC}^m &= \frac{-4Ab}{4b^2 + 9g^2 - 32}, \\
ed_{NC}^r &= \frac{-6Ag}{4b^2 + 9g^2 - 32}, \\
\eta &= 1/3, \\
\Pi_{NC}^m &= \frac{-4A^2}{4b^2 + 9g^2 - 32}, \\
\Pi_{NC}^r &= \frac{8A(8 - 3g^2)}{(4b^2 + 9g^2 - 32)^2}.
\end{align*}
\]

**Proof.** See Appendix B. ■

As expected, the findings in Proposition 2 indicate that the two players’ strategies and profits are positive and do not depend on the accrual rate when the cooperative advertising accrued budget is not restrictive. They are influenced by all the other three parameters, including the baseline demand \(A\), and the effects of both the retailer and manufacturer advertising \((g\) and \(b\)). Given the model’s assumptions and parameters’ ranges, these equilibrium strategies and outcomes are positive.

### 4.2 Cooperative advertising with accrual constraint

This subsection deals with scenarios where the manufacturer’s supported share of retailer’s advertising expenditures is greater than the accrued advertising budget, i.e., \(\min(rwd, \eta e_r^2) = rwd\). The equilibrium pricing and advertising decisions are denoted by the superscript \(C\) (Constraint) and are summarized in Proposition 3.

**Proposition 3** For the case where cooperative advertising is offered with an accrual constraint or when the accrual constraint is activated (i.e., \(rwd - \eta e_r^2 < 0\)), the equilibrium pricing and advertising decisions and profits for the two channel partners
are given by:

\[ w^C = \frac{A \left(g^2 - 4\right)}{(b^2 + 2g^2 - 8)(1 - r)}, \]

\[ p^C = \frac{3A \left(g^2 - 6\right)}{b^2 + 2g^2 - 8}, \]

\[ e^C_m = \frac{-Ab}{b^2 + 2g^2 - 8}, \]

\[ e^C_r = \frac{-Ag}{b^2 + 2g^2 - 8}, \]

\[ \Pi^C_m = \frac{-A^2}{b^2 + 2g^2 - 8}, \]

\[ \Pi^C_r = \frac{A^2 \left(4 - g^2\right)}{(b^2 + 2g^2 - 8)^2}. \]

**Proof.** See Appendix C. ♦

The findings in Proposition 3 indicate that all strategies and profits are positive given the model’s assumptions and parameters’ ranges. The wholesale price increases as the accrual rate increases at the equilibrium. On the other hand, all other channel decisions and profits are not directly affected by the accrual rate. Because the manufacturer’s budget allocation to cooperative advertising depends on their sales to the retailer, any increase of the accrual rate reduces their optimal profits if the wholesale price is kept constant. To ensure that the manufacturer’s profit remains unchanged when the accrual rate goes up, it is logical to increase the wholesale price. Manufacturers may therefore choose to offer generous cooperative advertising funds to retailers by setting wholesale prices that allow them to achieve better profitability.

## 5 Effects of the accrual constraint on strategies and profits

In the following three subsections, we compare the findings in Propositions 1-3 to assess the effects of the accrual constraint on equilibrium strategies and profits of the manufacturer and retailer. Recall that we respectively denote by \(NCA, NC,\) and \(C\) the game with no cooperative advertising, the game with cooperative advertising without accrual constraint, and the game with cooperative advertising and accrual constraint. Therefore the superscripts \(NCA, NC,\) and \(C\) are used to indicate the strategies and profits of the corresponding game.
5.1 Effects of the accrual constraint on advertising

Comparisons of the players’ advertising obtained in the three games lead to the findings summarized in Proposition 4.

**Proposition 4** When the accrual constraint is activated or not satisfied (i.e., $AC < 0$), the players’ advertising strategies in the three games compare as follows:

\[
\begin{align*}
\epsilon_m^{\text{NC}} & > \epsilon_m^{\text{C}} = \epsilon_m^{\text{NCA}}, \\
\epsilon_r^{\text{NC}} & > \epsilon_r^{\text{C}} = \epsilon_r^{\text{NCA}}.
\end{align*}
\]

**Proof.** Straightforward from comparisons of the advertising strategies obtained in Propositions 1, 2, and 3. ■

Proposition 4 supports the view that regardless of the value of the accrual rate, the manufacturer and retailer reduce their advertising investments when the accrual constraint is activated to the level where cooperative advertising is not offered. The retailer’s decision to reduce advertising investments due to the limited support from the manufacturer is expected. Unfortunately, the saving that the manufacturer realizes in setting a suboptimal cooperative advertising program does not translate to more manufacturer advertising. Zhang et al. (2015) also obtained a similar result, which can be better understood by further analyzing the effects of the accrual constraint on prices.

5.2 Effects of the accrual constraint on prices

Comparisons of the players’ prices for the games with and without the accrual constraint for the values of the parameters for which these two games are feasible and the accrual constraint is activated are summarized in Proposition 5 below.

**Proposition 5** When the accrual constraint is activated (or not satisfied), the comparisons of the equilibrium prices for the three games give the following results:

\[
\begin{align*}
w^{\text{NC}} & > w^{\text{C}} > w^{\text{NCA}}, \\
p^{\text{NC}} & > p^{\text{C}} = p^{\text{NCA}}.
\end{align*}
\]

**Proof.** Straightforward from comparisons of the pricing strategies obtained in Propositions 1, 2, and 3. ■

The findings in Proposition 5 indicate that when the retailer’s accrued cooperative advertising budget is not enough to support the retailer’s advertising expenses as
in the game without accrual constraint, channel members revert to price cuts to lessen the effects of suboptimal investments in cooperative advertising. Everything else being equals, a lower wholesale price for the manufacturer means a reduced margin that prevents him from reaching the optimal level of investment in his own advertising, as shown in Proposition 4. For the manufacturer, however, the price in the $C$ game is still higher than the price in the $NCA$ game in which a cooperative advertising program is not offered. The observed wholesale price increase in the $C$ game, compared to the $NCA$ game, helps the manufacturer to recoup the funds committed to support retailer advertising. In addition, the retail price is the same in both the $C$ and $NCA$ games. This is due to the fact that the manufacturer recoups the lump sum payment given as advertising support to the retailer via an increased wholesale price. As found in Proposition 4, the implication of this strategy is that the retailer maintains the same level of advertising in the $C$ and $NCA$ games because the accrued cooperative advertising budget in the $C$ game does not provide her with the necessary leverage to invest more in advertising the manufacturer’s product.

5.3 Effects of the accrual constraint on profits

The comparisons of the players’ profits in the three games are summarized in Proposition 6 below.

**Proposition 6** When the accrual constraint is activated (or not satisfied), the profits of the two players in the three games compare as follows:

\[
\begin{align*}
\Pi^N_{m} & > \Pi^C_{m} = \Pi^{NCA}_{m}, \\
\Pi^N_{r} & < \Pi^C_{r} = \Pi^{NCA}_{r}.
\end{align*}
\]

**Proof.** Straightforward from comparisons of profits obtained in Propositions 1, 2, and 3. $\blacksquare$

The manufacturer earns the same profit in both the $C$ and $NCA$ games, which is lower than the profit earned in the $NC$ game where a cooperative advertising program is offered without an accrual constraint. The retailer also earns the same profits in both the $C$ and $NCA$ games, but this time, this profit is higher than what she obtains in the $NC$ game. Thus, when the accrual constraint is activated, except for the wholesale prices, the strategies and profits of the two players are identical to those of the game without a cooperative advertising program. In addition to the fact that cooperative advertising funds received in a constrained program are not enough to pay for additional advertising expenses generated by such a program that
the retailer would not normally support in an NCA game, the manufacturer also sets an arbitrary cooperative advertising budget limit that is not directly related to the level of retailer advertising. As a matter of fact, channel members focus their advertising efforts on the availability of funds rather than on taking advantage of growth opportunities that might be created by higher advertising. This conservative approach leads basically to the status quo.

On the other hand, the preferences of the two players with respect to cooperative advertising programs with or without accrual constraint differ. A restrictive accrual constraint reduces (increases) the manufacturer’s (retailer’s) profits. Conversely, Zhang et al. (2015) found that the accrual constraint reduces both the manufacturer’s and retailer’s profits. The result for the retailer may seem counterintuitive in the sense that higher advertising investments in the NC game lead to a larger demand and can also lead to higher total channel profits compared to the C game. With an accrual constraint, the retailer does not effectively participate in the manufacturer’s cooperative advertising program as she maintains her status quo pricing and advertising strategies that keep the demand unchanged. This can be explained by the fact that the designed cooperative advertising is not attractive enough for the retailer at the status quo ($\Pi^{NC}_r < \Pi^{NCA}_r$) unless the manufacturer accepts to provide additional incentives to the retailer to increase her revenue or further reduce her advertising costs. The supply of additional incentives is possible not only because the manufacturer makes more profits with an unconstrained cooperative advertising ($\Pi^{NC}_m > \Pi^{C}_m$), but also due to the fact that the total channel profits can be higher. Therefore the manufacturer has the opportunity to make an unconstrained cooperative advertising program mutually beneficial with the retailer by sharing the cooperative advertising surplus. Examples of such incentives include price promotional activities targeted at retailers and various advertising support services offered to retailers. For example, Mitsubishi Motors’ cooperative advertising program is not limited to sharing dealers’ advertising costs as per the participation rate, it also includes an online advertising planner, which allows dealers to create their own ads without having to hire an advertising agency and to use pre-prepared layouts and creative copies (Jackson 2004). The use of this type of practices is increasing because the status quo is not a reasonable alternative for manufacturers in many industries.

6 Conclusion

Cooperative advertising programs are commonly offered by manufacturers to their retailers and generally contain two major components: a participation rate that
specifies the percentage of retailer advertising that a manufacturer is committed to pay and an accrual rate that specifies the maximum manufacturer’s contribution to the cooperative advertising program as a percentage of the retailer’s purchases from the manufacturer. This paper investigates in a bilateral monopoly context how the use of an accrual constraint in a cooperative advertising program affects channel members’ profits, as well as their pricing and advertising equilibrium decisions.

The main findings of this inquiry are as follows. Compared to the unconstrained cooperative advertising programs used in the literature, when an accrual constraint is included in the program and the accrual constraint is activated, meaning that the manufacturer’s contribution to the retailer’s advertising costs exceeds the accrued cooperative advertising budget, both the manufacturer and retailer reduce their advertising efforts to the level where cooperative advertising is not offered. The retailer also reduces the retail price to the level where cooperative advertising is not offered, while the manufacturer reduces the wholesale price, but keeps it higher compared to when cooperative advertising is not offered. These strategic moves translate to less (more) profits for the manufacturer (retailer).

The findings of this research show that pricing and advertising strategies recommended in the current cooperative advertising literature, which has mainly investigated unconstrained cooperative advertising programs (Aust and Buscher 2014; Jørgensen and Zaccour 2014), may not be optimal when manufacturers set limits to their cooperative advertising budgets based on retailers’ accrued purchases. Particularly, when the accrued budget prevents manufacturers from supporting the full share of the retailer’s advertising costs based on the participation rates, the retailer reverts to the status quo and maintains her pricing and advertising strategies of the game without cooperative advertising. The manufacturer slightly increases his wholesale price compared to when cooperative advertising is not offered, but this increase is not enough to stimulate more advertising investments. In such a context, the accrued cooperative advertising fund is mostly used as a side payment to compensate the retailer for smaller margins than an incentive to stimulate retailer advertising.

Compared to Zhang et al. (2015) who also study the use of accrual constraints, the current research shows that pricing decisions are key to understanding how channel members deal with accrual constraints in cooperative advertising programs. It therefore stresses the importance of studying the interplay between pricing and cooperative advertising decisions as advocated in previous works (Jørgensen and Zaccour 2014; Yan 2010). Unless the accrual constraint is not restrictive, i.e., it does not prevent the retailer from reaching her optimal advertising investments based exclusively on the participation rate, the inclusion of an accrual rate in a cooperative advertising program is counterproductive and should be avoid by manufacturers. As we already
know, as long as they are designed to align the interests of all channel members, unconstrained cooperative advertising programs give incentive to all parties involved to invest more in advertising and generate more profits that can be shared in various ways, including side payments and additional advertising support services.

Finally, there are several possible extensions of this work. Examples include the addition of long-term effects of advertising and competition either at the retail or manufacturing level.

References


7 Appendix

7.1 Appendix A: Equilibrium solutions for the scenario with no cooperative advertising (NCA game)

When there is no accrual constraint in the problem, the objective function for the retailer is \( \max_{p,e} \Pi_r = \max_{p,e} [(p-w)d-e^2_r] \). The Hessian matrix for the optimization
problem is
\[
\begin{bmatrix}
-2 & g \\
g & -2
\end{bmatrix}.
\]

To get a semi negative definite Hessian, the second-order conditions for the retailer are then:
\[
\begin{align*}
-2 & < 0, \\
4 - g^2 & > 0.
\end{align*}
\]

The first condition is always satisfied and the second condition is satisfied for \( g^2 < 4 \), which is also always true for \( g \in (0, 1) \) as per our model set-up. We then proceed to solving the retailer’s problem. Solving the first-order conditions, we obtain the retailer’s reaction functions which are given by:
\[
\begin{align*}
p &= \frac{g^2 w - 2 b e_m - 2 A - 2 w}{(g-2)(g+2)}, \\
e_r &= -\frac{g (b e_m + A - w)}{(g-2)(g+2)}.
\end{align*}
\]

We then replace the retail price and retail advertising by the obtained expressions above in the manufacturer’s profit function and solve the manufacturer’s problem given by \( \max_{w,e_m} \Pi_m = \max_{w,e_m} [wd - e_m^2] \). The Hessian for the manufacturer’s problem is
\[
\begin{bmatrix}
\frac{4}{(g-2)(g+2)} & -\frac{2b}{(g-2)(g+2)} \\
-\frac{2b}{(g-2)(g+2)} & -\frac{2g^2 - 8}{(g-2)(g+2)}
\end{bmatrix}.
\]

The second-order conditions for the manufacturer are
\[
\begin{align*}
g^2 - 4 & < 0, \\
b^2 + 2g^2 - 8 & < 0.
\end{align*}
\]

Again, for \( g, b \in (0, 1) \), both the first and the second conditions are true. We then solve the first-order conditions of the manufacturer’s problem to get the equilibrium manufacturer’s wholesale price and advertising strategies. The results are as following:
\[
\begin{align*}
w &= \frac{A (g - 2) (g + 2)}{b^2 + 2g^2 - 8}, \\
e_m &= -\frac{bA}{b^2 + 2g^2 - 8}.
\end{align*}
\]
Now, we substitute the obtained equilibrium expressions of $w$ and $e_m$ into the retailer’s reaction functions previously obtained, and get the equilibrium solution for the basic problem without any cooperative advertising such as

$$p = \frac{A (g^2 - 6)}{b^2 + 2g^2 - 8},$$

$$e_r = \frac{-Ag}{(b^2 + 2g^2 - 8)},$$

$$\Pi_r = \frac{-A^2 (g - 2) (g + 2)}{(b^2 + 2g^2 - 8)^2},$$

$$\Pi_m = \frac{-A^2}{b^2 + 2g^2 - 8},$$

$$d = \frac{-2A}{b^2 + 2g^2 - 8}.$$

In the paper, these strategies are denoted by the superscript $NCA$. For $A > 0$, $g, b \in (0, 1)$ we verify that the following conditions are satisfied at the equilibrium: the retailer’s pricing, advertising effort and profit are positive and the retail price is higher than the wholesale price to guarantee positive retail margins. We also verify that the manufacturer’s wholesale price, advertising effort and profit are positive. Finally, we verify that the demand is positive. As previously stated, the second-order conditions for all problems are also verified for the parameters’ ranges specified in the model.

### 7.2 Appendix B: Equilibrium solution for the scenario with cooperative advertising and no accrual constraint (NC game)

In this case, the manufacturer reimburses a percentage $\eta \in (0, 1)$ of the retailer’s advertising cost. We solve for the equilibrium solution by backward induction and start by the retailer’s problem. The latter is given by max$_{p,e_r} \Pi_r = max_{p,e_r} [(p - w)d - (1 - \eta) e_r^2]$. Given the retailer’s profit function in this scenario, the Hessian matrix of the optimization problem (with respect to retailer’s price and advertising cost for the manufacturer, i.e., $p, e_r$) is as following:

$$\begin{bmatrix}
-2 & g \\
g & 2(\eta - 1)
\end{bmatrix}.$$
To ensure a semi negative definite Hessian, the second-order conditions for the retailer are then:

\[-2 < 0,\]
\[4 (1 - \eta) - g^2 > 0.\]

The first condition is always satisfied. The second condition is satisfied iff \(\eta < (1 - \frac{g^2}{4})\). We then proceed to solving the retailer’s problem and discuss the feasibility of the second condition later at the point of optimal \(\eta\). Solving the first-order conditions of the retailer’s problem, we obtain the retailer’s reaction functions which are given by:

\[p = \frac{(2 be_m + 2 A + 2 w) \eta + g^2 w - 2 be_m - 2 A - 2 w}{g^2 + 4 \eta - 4},\]
\[e_r = \frac{-g (be_m + A - w)}{g^2 + 4 \eta - 4}.\]

Next, we replace the retailer’s price and advertising by the obtained expressions in the manufacturer’s profit functions and solve the manufacturer’s problem given by \(\max_{w,\eta,e_m} \Pi_m = \max_{w,e_m} [wd - e^2_m - \eta e^2_r]\).

The second-order conditions for the manufacturer are

\[-8\eta^2 - (3g^2 - 16) \eta - 2 (4 - g^2) < 0,\]
\[\left(b^2 - 8\right) \eta^2 + (16 - 2b^2 - 3g^2) \eta + \left(b^2 + 2g^2 - 8\right) < 0,\]
\[f(e_m, \eta, w) < 0.\]

where \(f(e_m, \eta, w)\) is a polynomial of degree 2 and is too long to include here. We discuss the validity of these conditions at the point of optimal \(\eta\) later in this section. We solve the first-order conditions of the manufacturer’s problem to get the equilibrium manufacturer’s wholesale price and advertising effort. The results are as follows:

\[w = \frac{A (3 g^2 - 16)}{4 b^2 + 9 g^2 - 32},\]
\[e_m = -\frac{4 b A}{4 b^2 + 9 g^2 - 32},\]
\[\eta = \frac{1}{3}.\]
Next, we substitute $w$, $\eta$ and $e_m$ into the retailer’s reaction functions, and obtain the equilibrium solution for the unconstrained problem such as

$$p = \frac{3(g^2 - 8)A}{4b^2 + 9g^2 - 32},$$

$$e_r = \frac{-6Ag}{4b^2 + 9g^2 - 32},$$

$$\Pi_r = \frac{-8A^2(3g^2 - 8)}{(4b^2 + 9g^2 - 32)^2},$$

$$\Pi_m = \frac{-4A^2}{4b^2 + 9g^2 - 32}.$$

In the paper, these strategies are denoted by the superscript $NC$. For $A > 0$, $g, b \in (0, 1)$ we verify that the following conditions are satisfied at the equilibrium: the retailer’s pricing, advertising effort and profit are positive and the retail price is higher than the wholesale price to guarantee positive retail margins. We also verify that the manufacturer’s wholesale price, advertising effort and profit are positive and that the coop advertising rate ($\eta$) is between 0 and 1. Finally, we verify that the demand is positive at the equilibrium. The second-order conditions for the retailer’s problem are verified iff $g^2 + 4\eta - 4 < 0$. Therefore, at the equilibrium ($\eta = \frac{1}{3}$), this condition is satisfied for any $g \in (0, 1)$. Similarly, all second-order conditions for the manufacturer’s problem are always verified at the equilibrium for $g, b \in (0, 1)$.

### 7.3 Appendix C: Equilibrium solution for the scenario with cooperative advertising and the accrual constraint (C game)

When an accrual constraint ($AC$) is employed by the manufacturer a part of the cooperative advertising program, two scenarios can occur. Given a specific accrual rate $r$, when the accrual constraint is satisfied, i.e., $rwd - \eta e_r^2 \geq 0$, the channel members will choose the equilibrium solution for the NCA game. Replacing $w, d, \eta$ and $e_r$ by their equilibrium expressions in the NC game ($rw^{NC}d^{NC} - \eta^{NC}(e_r^{NC})^2 \geq 0$), this leads to the following constraint

$$r \geq \frac{3g^2}{2(16 - 3g^2)}.$$
A representation of this constraint is shown in Figure A.1. For a specific value of the parameter $g$, the accrual constraint represented by the curve in Figure 1 is satisfied iff the accrual rate $(r)$ is higher than the corresponding value on the curve. Therefore, in the area above the curve, the manufacturer and the retailer will choose the equilibrium in the NC game. However, in the area below the curve, i.e., for lower values of the accrual rate $(r)$, the equilibrium for the NC game cannot be implemented. Instead of paying an amount of $\eta e_r^2$ to the retailer, the manufacturer pays only an amount corresponding to $rwd$. We need then to solve the retailer’s and the manufacturer’s problems to find the equilibrium solution for values of $r$ and $g$ that are below the curve in Figure A.1.

![Figure A.1. The accrual constraint](image)

When the constraint is not satisfied (or active) i.e., $r < \frac{3g^2}{2(16-3g^2)}$, the retailer’s problem is $\max_{p,e_r} \Pi_r = \max_{p,e_r} [(p-w)d-e_r^2 + rwd]$. The Hessian matrix associated with this problem is given by:

$$
\begin{bmatrix}
-2 & g \\
g & 2
\end{bmatrix}
$$

The second-order conditions for the retailer are:

$$
-2 < 0, \\
4 - g^2 > 0.
$$
Both of these constraints are satisfied for $g \in (0, 1)$. We then proceed to solving the retailer’s problem by solve the first-order conditions to get the retailer’s reaction functions:

\[
p = -\frac{(g^2 - 2)(r - 1)w - 2bem - 2A}{(g - 2)(g + 2)},
\]
\[
e_r = -\frac{g((r - 1)w + bem + A)}{(g - 2)(g + 2)}.
\]

The manufacturer’s problem in this scenarios is given by $\max_{w, e_m} \Pi_m = \max_{w, e_m} [wd - e_m^2 - rwd]$. We plug the retailer’s reaction functions into the manufacturer’s profit and get the following expression

\[
\Pi_m = \frac{2w(r - 1)(A + bem) + e_m^2(4 - g^2) + 2w^2(1 - r)^2}{(g - 2)(g + 2)}.
\]

The Hessian matrix for the manufacturer is given by:

\[
\begin{bmatrix}
\frac{4(r^2 - 1)^2}{(g - 2)(g + 2)} & \frac{2br - 2b}{(g - 2)(g + 2)} \\
\frac{2br - 2b}{(g - 2)(g + 2)} & \frac{-2g^2 + 8}{(g - 2)(g + 2)}
\end{bmatrix}.
\]

The second-order conditions for the manufacturer are then

\[
\frac{4(r - 1)^2}{(g - 2)(g + 2)} < 0,
\]
\[
\frac{-4(r - 1)^2(b^2 + 2g^2 - 8)}{(g - 2)^2(g + 2)^2} > 0.
\]

Both of these conditions are always verified for any $g, b, r \in (0, 1)$.

Next, solving the first-order conditions for the manufacturer’s problem, we get the following results:

\[
w = \frac{A(g - 2)(g + 2)}{(b^2 + 2g^2 - 8)(1 - r)},
\]
\[
e_m = \frac{-bA}{(b^2 + 2g^2 - 8)}.
\]
Next, we plug the obtained \( w \) and \( e_m \) into the retailer’s reaction functions and obtain the following equilibrium solution:

\[
\begin{align*}
p &= \frac{A (g^2 - 6)}{b^2 + 2g^2 - 8}, \\
e_r &= \frac{-Ag}{b^2 + 2g^2 - 8}, \\
\Pi_m &= \frac{-A^2}{b^2 + 2g^2 - 8}, \\
\Pi_r &= \frac{-\left(g - 2\right) \left(g + 2\right) A^2}{(b^2 + 2g^2 - 8)^2}, \\
\Pi_{ch} &= \frac{-A^2 \left(b^2 + 3g^2 - 12\right)}{(b^2 + 2g^2 - 8)^2}, \\
p - w &= \frac{A \left(g^2 r - 6r + 2\right)}{(b^2 + 2g^2 - 8) (r - 1)}.
\end{align*}
\]

In the paper, these strategies are denoted by the superscript \( C \). For any \( A > 0, \ g, b, r \in (0, 1) \) we verify that the following conditions are satisfied at the equilibrium: the retailer’s pricing, advertising effort and profit are positive and the retail price is higher than the wholesale price to guarantee positive retail margins. We also verify that the manufacturer’s wholesale price, advertising effort and profit are positive. Finally, we verify that the demand is positive at the equilibrium and that second order conditions for all games are true.