Design Synthesis of Car-Trailer Systems with Active Trailer Differential Braking Strategies

by

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AUTHOR’S DECLARATION

I hereby declare that I am the sole author of this thesis. This is a true copy of the thesis, including any required final revisions, as accepted by my examiners.

I understand that my thesis may be made electronically available to the public.
ABSTRACT

To this date, various control strategies based on linear vehicle models have been proposed and developed for improving the lateral stability of car trailer (CT) systems. Is a linear-model-based controller applicable to active safety systems for CT systems under emergency operating conditions, such as an evasive maneuver at high lateral accelerations? In order to address the problem, the following innovative investigations have been conducted: 1) a comparative study of typical linear and nonlinear CT models have been carried out to examine the dynamic responses of the models under the emulated test maneuvers; and 2) the applicability of an Active Trailer Differential Braking (ATDB) controller designed using a linear CT model is tested and evaluated under the conditions that the controller is applied to a CT system represented by the selected linear and nonlinear models. The current research leads to the following insightful findings: 1) the selected linear CT model is effective to predict the lateral stability of CT systems; 2) under the regular evasive maneuvers at low lateral accelerations (less than 0.5g), this linear model can be used to provide dynamic responses that are in good agreement with the selected nonlinear models; 3) the ATDB controller
designed using the linear model can effectively improve the lateral stability of CT systems under regular evasive maneuvers at low lateral accelerations, but the controller is not applicable to CT active safety systems under emergency evasive maneuvers at high lateral accelerations. The insightful findings resulted from the thesis will provide valuable design guidelines for the development of active safety systems for CT systems.
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DEDICATION

I would like to thank also my mom and dad for their unconditional love, affection and support in my life.
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## NOMENCLATURE

- $C_f$: Cornering stiffness of the front tire of tractor
- $C_r$: Cornering stiffness of the rear tire of tractor
- $C_t$: Cornering stiffness of the tire of trailer
- $X_{\psi_2}$: Longitudinal hitch reaction force of tractor
- $Y_{\psi_2}$: Lateral hitch reaction force of tractor
- $Y_f$: Lateral force acted on the front axle of tractor
- $Y_r$: Lateral force acted on the rear axle of tractor
- $Y_t$: Lateral force acted on the front axle of trailer
- $I_c$: Principal yaw mass moment of inertia of tractor
- $I_t$: Principal yaw mass moment of inertia of trailer
- $m_c$: Total mass of tractor
- $m_t$: Total mass of trailer
- $a$: Distance between front axle and centre of gravity (CG) of tractor
- $b$: Distance between rear axle and CG of tractor
- $d_1$: Distance between hitch and rear axle of tractor
- $d_2$: Distance between tire of trailer and hitch
- $c$: Distance between trailer axle and CG of trailer
- $U$: Longitudinal speed of tractor
- $U_t$: Longitudinal speed of trailer
- $V$: Lateral speed of tractor
- $V_t$: Lateral speed of trailer
- $\alpha_f$: Side-slip angle of front tire of tractor
- $\alpha_r$: Side-slip angle of rear tire of tractor
- $\alpha_t$: Side-slip angle of tire of trailer
- $\delta_f$: Steering angle of front tire of tractor
\( \dot{\psi}_1 \) Yaw rate of tractor
\( \dot{\psi}_t \) Yaw rate of trailer
\( \psi_2 \) Articulation angle between tractor and trailer
\( \phi_1 \) Roll angle of tractor
\( \phi_2 \) Roll angle of trailer
\( T \) The track between the left and right wheels of trailer
\( H_H \) The vertical distance from car body centre of mass to hitch
\( H_{TC} \) The vertical distances to the roll center of the trailer suspension respectively
\( P_l \) The longitudinal position of the center of trailer wheels, which are respect to the hitch point for the trailer
\( P_j \) The lateral position of the center of trailer wheels
\( P_k \) The vertical position of the center of trailer wheels
Chapter 1

INTRODUCTION

1.1 CAR-TRAILER SYSTEMS’ CONFIGURATIONS

A car-trailer system generally consists of a powered unit, such as a SUV or passenger car, and a towed unit, the trailer. Individual units are connected at an articulated point by a hitch [1]. Figure 1.1 shows a common passenger car and a trailer used in CT (car-trailer) system.

![Car-trailer systems’ configurations](image)

Figure 1.1 Car-trailer systems’ configurations

1.2 WHY ARE CAR-TRAILER SYSTEMS WIDELY USED?

In North America, car-trailer systems are widely used to transport goods and materials because CT systems are cost-effective and versatile in the daily life of most families [1]. Compared to passenger cars, the CT system can transport more freight and have better
fuel economy. Moreover, as an independent unit of the CT system, the trailer can be readily connected and disconnected by the owner at anytime which helps the owner choose the suitable type of trailer for his/her actual needs. Given all these advantages, CT systems are becoming the preferred mode of transportation for North America families.

1.3 MOTIVATIONS

Nowadays, considering the ever-increasing density of highway traffic due to the complex structures of the CT system and relatively high average transport velocity, the traffic safety or the stability of car-trailer systems has become an important issue. In the US, more than 35,000 people were killed in road accidents each year from 1993 to 1998 [2].

Road vehicle safety or stability depends on the characteristics of vehicle motions, such as longitudinal motion, lateral motion, and vertical motion. The lateral stability is directly related to the safety of road vehicles and it has become an important research topic in vehicle dynamics. For a single unit vehicle, unstable lateral motion may result in spinouts and rollover accidents. As articulated vehicles, CT systems have more complex structures than single unit vehicles; therefore, CT systems have unique unstable motion modes which include the jack-knifing, trailer swing, and rollover [3].

As the first type unstable motion, jack-knifing is one of the most common causes for serious traffic accidents in which the car and trailer are involved. It is mainly attributed to tire/ground friction force saturation that may occur in curved path negotiations or during
heavy braking processes. If the articulation angle between the leading and trailing units exceeds a critical limit, the driver is unable to control the motion of the vehicle by steering. Figure 1.2 shows the jack-knifing of a CT system.

![Figure 1.2 Schematic presentation of the jack-knifing of a CT system.](image)

The second type of unstable motion mode is the trailer sway. Similar to the jack-knifing, the trailer sway is also a yaw instability mode in term of divergent trail yaw response. This unstable motion mode usually occurs when the side forces on the trailer cause the trailer to move side to side behind the tow vehicle. Two factors are involved here: (1) the side forces on the trailer, which are caused by towing speed, gusting winds, bow ware, bad roads, and downhill travel; (2) the location and the type of hitch where the trailer is linked to the tow vehicle, such as poor trailer design and poor hitch adjustment [4]. Figure 1.3 shows the trailer sway of a CT system.

Figure 1.3 Schematic presentation of the trailer sway of a CT system.
The last type of unstable motion mode is roll-over. Three major factors contributing to rollover accidents are: 1) high-speed curved path negotiations, such as a CT system operating at a high merging speed on highway ramps; 2) sudden course deviation from high initial speed, e.g., lane-change maneuvers; and 3) load shift. The rollover stability is limited by the vehicle’s static rollover threshold expressed as the lateral acceleration in gravitational units (g). Single unit car rollover thresholds are higher than 1g, while the threshold of a car-trailer system may be as low as 0.6 g [5]. The unstable motion modes may lead to fatal accidents which has been a significant problem in terms of both social and economic cost [23, 25]. Figure 1.4 shows the trailer roll-over of a CT system.
Figure 1.4 Schematic presentation of the trailer roll-over of a CT system.

To the date, the majority of car-trailer systems use passive mechanisms to enhance the lateral stability at high speeds. Unfortunately, the high-speed lateral stability of a CT system cannot be guaranteed with a passive mechanism because the operation conditions vary significantly, such as driving style, load, road and weather conditions [6].

To address the safety issues of CT systems, various active safety systems have been proposed, which include active trailer steering and active trailer braking [4]. In order to develop an effective CT active safety system, it is critical to design a robust control algorithm that prevents the unstable motion modes. In the initial development phase of road vehicle active safety systems, various vehicle models have been used to derive the corresponding control algorithms. In the controller design for articulated vehicle active
safety systems, different vehicle models with various degrees of freedom (DOF) have been used [3, 6, and 7]. However, the applicability of various articulated vehicle models for the active safety systems design has not been addressed in the literature. This provides compelling motivation to examine the dynamic behaviors of the typical CT models and to design a new active trailer differential braking (ATDB) strategy to improve the lateral stability of CT systems.

1.4 THESIS CONTRIBUTIONS

A linear 3-DOF, a nonlinear 4-DOF, and a nonlinear 6-DOF CT models are modeled, compared, evaluated, and validated using a CT model developed with the commercial software package, CarSim. A design method is proposed for the design of ATDB systems for CT systems in order to improve the lateral stability of CT systems. The new ATDB strategy has been designed for the nonlinear 4-DOF, the nonlinear 6-DOF, and the nonlinear 21-DOF CarSim CT models using a set of control gains which are based on a 3-DOF linear model and the linear controller (LQR) technique. Moreover, to demonstrate the effectiveness of the new ATDB strategy, numerical simulations have been conducted using Matlab/Simulink and CarSim software.

1.5 THESIS ORGANIZATION

The thesis is organized as follows: Chapter 1 provides general background information of CT systems and three unique unstable motion modes of CT systems. Chapter 2 presents a comprehensive literature review on research related to articulated vehicle
dynamics and also on the state-of-the-art of CT passive and active safely control systems. Chapter 3 introduces the relevant vehicle models: CarSim was used generated, compared, evaluated and validated a CT model, then using the linear 3-DOF, the nonlinear 4-DOF and nonlinear 6-DOF CT models. Chapter 4 presents the ATDB system for CT systems. The new ATDB strategy is designed for the nonlinear 4-DOF, the nonlinear 6-DOF and the CarSim CT models by using a set of control gains based on the 3-DOF linear model and the LQR technique. Chapter 5 concludes the thesis, summarizing the results and providing suggestions for future work in the field of the current research.
Chapter 2

LITERATURE REVIEW

2.1 INTRODUCTION

Nowadays, considering the ever-increasing density of highway traffic and the complex structures along with the relatively high average transport velocity of CT systems, the traffic safety and stability of CT systems has become an important issue.

The objective of this chapter is to conduct a comprehensive literature review on the research related to articulated vehicles dynamics and also on the-state-of-the-art of CT passive and active safety control system design methodologies. We also point out research areas where significant future contributions to the field can be made.

2.2 A BRIEF HISTORY OF ARTICULATED VEHICLE DYNAMICS

Road vehicle safety and stability depends on the characteristics of vehicle dynamics such as the lateral stability, maneuverability and handling performance. For a single unit vehicle, unstable lateral motions and vertical motions may result in a spin-out or a rollover accident. Articulated vehicle dynamics is a natural expansion and extension of single vehicle dynamics. As an articulated vehicle, a CT system has more complex structures than a single unit vehicle, so they have unique unstable motion modes which include the jack-knifing, trailer swing, and rollover [3, 48].
Over the past two decades, numerous simulation-based theoretical studies have been published. Experimental analysis of CT stability has also been studied in order to develop safety control systems and improve the lateral stability of articulated vehicle.

### 2.2.1 Lateral Stability of Articulated Vehicles

Several studies have been conducted to analyze the risk of instability in the yaw dynamics of articulated vehicles in the past two decades. In this regard, a latest literature survey provides substantial references concerning the lateral dynamics of articulated vehicles by Vlk [8]. In this literature survey, three lateral instability motions are presented and analyzed: jack-knifing, trailer swing, and an oscillatory motion of the towed unit.

The trailer swing is defined as the lateral high speed oscillation of a towed vehicle at high vehicle speeds [13]. The oscillations occur at a typical frequency of 0.6 Hz, and the amplitude will increase with the forward speed or the mass ratio of trailer to car [9]. D. Fratila generated a mathematical model of a car/caravan system with twenty-four degrees of freedom. In this model suspension and tire flexibility were considered [20]. It has been shown that the most important influences of the lateral stability of caravans are: speed, caravan mass, caravan yaw inertia, towed trailer load, axle position, wheel track and tire cornering stiffness.

Jack-knifing is defined as a loss of yaw stability of articulated vehicles [10-12]. It is mainly attributed to tire/ground friction force saturation that may occur in curved path
negotiations or during heavy braking process. If the articulation angle between the leading and trailing units exceeds a critical limit, the driver will be unable to control the motion of the vehicle by steering. This unstable motion most commonly occurs when the trailer is empty.

These unstable motions cause the vast majority of loss-of-control accidents. According to the study reported by the National Highway Traffic Safety Administration [24], in the United States in 2003, 3.1% of all examined crashes caused by tractor-trailer combinations are due to a jack-knifing event.

2.2.2 Roll Stability of Articulated Vehicles

Roll stability is related to the possibility that the vehicle will roll over [21]. Due to more complex structures, an articulated vehicle has lower roll stability than a single unit vehicle, especially on inclined grounds and banked roads at low speeds [22]. D. Fratila devised a mathematical model of the car/caravan system with twenty-four degrees of freedom. In this model, caravan suspension and tire flexibility are considered [20]. It has been shown that the most important influences of the lateral stability of caravans are: caravan suspension damping, caravan roll inertia and the height of the centre of gravity.

2.3 SAFETY CONTROL SYSTEMS OF ARTICULATED VEHICLES

Nowadays, safety has been a common concern for all roads users, especially due to the ever-increasing density of highway traffic. Over the years, many control systems for
reducing trailer swing, jack-knifing, and rollover instability have been developed, which include passive control [14-19] and active control [26-32] strategies.

### 2.3.1 Passive Stability Control Systems

Over the last two decades many articulated vehicles passive stability control systems have been investigated by researchers [14-19]. The results have shown that three major unstable motions of articulated vehicles depend on the parameters of the towing vehicle, the vertical hitch load and towed trailer. More specifically, the jack-knifing depends on the trailer mass and the location of the trailer center of gravity in the longitudinal direction. Therefore, in order to reduce the influences of the jack-knifing motion, a commonly used method is to reduce the hitch load or move the trailer’s center of gravity rearward. The snaking motion depends on the parameters of both the towing vehicle and the trailer. According to the study by Bevan [18], some passive control methods have been investigated, such as: 1) to increase the mass of the trailer; 2) to move rearward the center of gravity of the trailer; 3) to increase the moment of inertia of the trailer; 4) to decrease cornering stiffness of trailer tires; 5) to decrease cornering stiffness of the towing unit’s rear tires; 6) to increase the distance from the vehicle rear axle to the hitch point; 7) to increase vehicle wheelbase.

### 2.3.2 Active Stability Control Systems

Unfortunately, the high-speed lateral stability of an articulated vehicle system cannot be guaranteed with a passive mechanism, because the operation conditions vary significantly,
i.e., different drivers, load, road, tire type and tire pressure and weather conditions [6]. To address the safety problem of CT systems, various active safety systems have been proposed, which include active four-wheel steering of the towing vehicle, active trailer steering and active trailer braking [4].

2.3.2.1 Active Steering Strategy

Shuwen [33] presented an investigation to enhance the lateral stability of the CT combination by using four-wheel steering system. In this paper, a 4-DOF CT combination model was used as the controller predictive model. Based on a sliding mode control, a four-wheel steering system is designed to control the rear wheel steering angle, which can control the yaw rate and the side slip angle. The results show that this four-wheel steering system clearly improves the lateral stability and active safety of the CT combination.

Islam and Yuping [34] examined and evaluated an active trailer steering (ATS) system to improve maneuverability at low speeds and enhance lateral stability at high speeds of articulated heavy vehicles (AHVs), which is based on an automated design synthesis approach. In this method, a driver model is developed, two operational modes are designed and one optimization control method is designed using Genetic Algorithms. Simulation results indicate that the design-based optimization of AHVs with ATS system is an effective method to improve the lateral stability at high speeds, and enhance the path-following performance at low speeds.
2.3.2.2 Active trailer differential braking strategy

Active trailer differential braking strategy is used by some researchers [35, 36] to control the lateral stability at high speeds by preventing unstable motion modes, such as trailer swing, jack-knifing, and rollover. This strategy controls brake pressures on each tire of the same trailer axle. It can produce a yaw movement by using tire differential braking forces, which can control the yaw motion of the trailer in order to improve the stability of articulated vehicles. In order to derive this theory, Peng [37] modeled the brake line pressure of tire brake force subsystem.

Li and Pu [38] presented the differential braking control algorithm to improve the yaw/rollover stability of tractor semi-trailers. This strategy is based on a PID controller and the control algorithms in MATLAB. The simulation results show that the differential braking control algorithm can not only improve the yaw stability, but also improve the rollover stability and greatly improve the track holding ability.

Tom [39] devised a brake steer system which used differential brake forces for steering intervention in the context of Intelligent Vehicle Highway Systems. This system was implemented by a state feedback regulator and PID controller. It also presented a way to implement brake steer moment by adding an external differential brake moment. The simulation results have shown that the brake steer system provides steering intervention under maneuvers in order to avoid road departure.

Charles and Robert [41] presented a report on active safety systems for limited authority
lateral maneuvering with a differential braking system. In this report, the researcher expounded the basic operation principle of the differential braking system. This control algorithm can change the direction of a vehicle when it is going away from the roadway through intelligently adjusted differential braking forces on the left or right-side wheels. Of course, the traditional control channel can still be used by drivers to steer the vehicle when the vehicle is steered by utilizing braking, because the original steering system is not interfered with or modified using this approach. The results have shown that the algorithm can be adapted to accommodate a wide range of operating conditions and roadway-departure scenarios for improving stability of vehicle and road safety.

AL-KO Kober AG Company [40] presented a new approach: the oscillations were sensed directly from the trailer. The combination of the vehicle and trailer are stabilized by an active trailer differential braking controller. The AL-KO system’s specific operation process is shown in Figure 2.1.
When the trailer yaw rate stability sensor detects some emergency situation (e.g. the trailer is about to lose stability), the differential braking system will instantly produce the brake forces on both trailer wheels: The brake force will firstly work on the wheel, which opposite to the direction of sway, harder than the other, thus pulling the trailer into line. Then brake forces intensity is increased and applied equally on both wheels to keep the car and the trailer into one line [40]. The result has shown that this braking strategy is effective to improve the trailer swing at high speed.

2.4 TIRE MODELS

With the development of vehicle dynamics, researchers have found the special importance of tire force and moment characteristics in vehicle dynamic studies. As the
only relation between the vehicle and the road surface, the tire characteristics ultimately
determine the driving characteristics, stability of the vehicle and the ride comfort. At the
present time there are many different tire models for vehicle dynamics analysis such as
the TMeasy tire model, the TreadSim tire model, the dynamic tire friction model of Deur
and the Magic Formula tire [60]. One of the most important tire models is the Magic
Formula tire model by Pacejka [47], which is widely used to calculate steady-state tire
force and moment characteristics for vehicle dynamic studies. The development of the
model was started in the mid-eighties. Some characteristics analysis and studies of the
tire model have been developed in cooperation between the TU-Delft and Volvo, and
these results are presented in the literature [42], [43] and [46]. A simple analysis method
is introduced to describe the tire horizontal force generation at combined slip of the
Magic Formula, which is based on weighting functions [44]. A more detailed description
of the Magic Formula tire model is given in the book of Pacejka [45].

2.5 MATHEMATICAL VEHICLE MODEL

Over the last two decades, a large number of mathematical vehicle models have been
developed in order to better understand the high speed stability of towed vehicles [55, 61].
And more complete dynamic factors of the CT model are considered in these researches.

A car-trailer model with 3-DOF was used by Bevan [15] and Deng and Kang [19] in their
research. In this model, the car and trailer are joined at the hitch point and assume a
constant forward speed together with the yaw–side-slip degree of freedom.
Anderson and Kurtz [16] developed both a 4-DOF model and a 6-DOF model in order to study longitudinal dynamics of both models and the roll dynamics of both the car and the trailer.

Fratila and Darling [14] developed a more comprehensive CT model with 24-DOF. This model included all yaw, pitch, and roll motions of both the vehicle and the trailer. The unsprung mass vertical and spin motions were also considered in this model.

With the wide range of model development, commercial multibody dynamics simulation software has also been used in vehicle system investigations to generate complex nonlinear vehicle models with many degrees of freedom, such as CarSim, TruckSim, ADAMS and DADS [49-54]. For CT system, Sharp and Fernandez [13] developed a highly sophisticated 32-DOF car–caravan model using AutoSim.

In this thesis, a linear 3-DOF, a nonlinear 4-DOF, a nonlinear 6-DOF and a CarSim-based CT model were developed. They are used to study the dynamic characters and design control system for CT system. These models include the necessary vehicle states while neglecting some of the irrelevant states.

2.6 COMPUTER SIMULATION OF VEHICLE DYNAMICS

With the development of vehicle dynamics and computer simulation technology, many controller design methods have been widely used to study the dynamic characteristics and design control systems, such as LQR controller, fuzzy logic controller [56, 57] and
proportional-integral-derivative (PID) controller [63]. One of the most important controller design methods is the LQR control technique.

Recently, in order to improve both low-speed maneuverability and high-speed stability of multi-trailer articulated heavy vehicles, active trailer steering systems were design using the LQR controllers [58, 59].

Tianjun and Changfu [62] presented the differential braking strategy for a heavy tractor semi-trailer, which is based on a six-axle plane dynamic model with 4-DOF. A LQR controller was designed to improve the yaw-roll stability of the tractor semi-trailer and a PD controller was designed to regulate the longitudinal slip ratio of tires. The simulation results showed that yaw-roll stability of the tractor semi-trailer had improved effectively after applying the differential braking.

2.7 LIMITATIONS OF EXISTING DESIGN AND ANALYSIS METHODS

To date, most researchs focused on studying the effects of either passive or active trailer control systems based on dynamic simulations and analysis. Overall, these strategies are based on two basic underlying vehicle models, i.e., linear models and nonlinear models, and corresponding control techniques. However, these design strategies have some limitations. In these studies, the LQR adopted the linear model. If one strategy is based on a nonlinear model, then the strategy must adopt a complex nonlinear controller.
However, in the real case, the nonlinear controller has more complex structure and higher costs.

Therefore, in order to address the limitations of the existing design and analysis approaches, this thesis will investigate innovative methods for the design of CT systems with ATDB systems, which can improve the lateral stability of CT system at high speed.

**2.8 RESEARCH OBJECTIVES**

The main objectives of this thesis are as follows:

1) To generate, compare, evaluate and validate a linear 3-DOF, a nonlinear 4-DOF and a nonlinear 6-DOF model using a CT model developed with the commercial software package, CarSim.

2) To design an ATDB controller in order to increase the stability, and consequently the safety level of CT systems.
Chapter 3

VEHICLE MODELING AND EVALUATION

3.1 INTRODUCTION

This thesis will examine and evaluate an ATDB strategy to improve the lateral stability of CT systems. This chapter introduces the relevant vehicle models: a linear 3-DOF, a nonlinear 4-DOF, and a nonlinear 6-DOF CT models, which are generated, compared, and evaluated using a nonlinear CarSim model with 21-DOF. The comparative study of the CT model is implemented through investigating numerical simulation results obtained in an emulated test, i.e., a single lane-change maneuver. The deviations of the model dynamic responses are discussed [71].

3.2 TIRE MODEL

In this thesis, the simple Magic Formula tire model [46] is selected to model the tire/road force, and the impact of camber is ignored.

In the specific description, the tire lateral force is defined as a function of normal load and tire side-slip angle as shown in Eq. (3.1). Here, $x$ represents the side-slip angle, and $y$ represents the corresponding lateral forces. The typical curve of the Magic Formula tire model is shown in Figure 3.1
The classic curve of the Magic Formula tire model [46]

The coefficients $B, C, D, E$, are curve fitted parameters and shown in the Nomenclature. In Figure 3.1, the curves describe the relationship between tire force and the corresponding tire slip parameters at a given vertical load [45]. In the full formulation, these curve fitted parameters can be defined as functions of vertical load and tire side-slip angle as shown in Eqs. (3.2) - (3.5). The corresponding coefficient values are shown in Table 3.1. Note that all side-slip angles are limited to 90 degrees in this thesis.

\[
y(x) = D \cdot \sin \left[ C \cdot \arctan \left( B \cdot x - E \cdot (B \cdot x - \arctan (B) \cdot x) \right) \right]
\]  

(3.1)

\[
D = a_1 \cdot F_z^2 + a_2 \cdot F_z
\]  

(3.2)
\[ C = a_0 \]  

(3.3)

\[ B \cdot C \cdot D = a_3 \cdot \sin(2 \cdot \tan^{-1}\left(\frac{F_z}{a_4}\right)) \text{ Lateral force} \]  

(3.4)

\[ E = (a_6 \cdot F_z + a_7) \]  

(3.5)

Table 3.1. The corresponding general coefficients of the Magic Formula tire model

<table>
<thead>
<tr>
<th>(a_0)</th>
<th>(a_1)</th>
<th>(a_2)</th>
<th>(a_3)</th>
<th>(a_4)</th>
<th>(a_5)</th>
<th>(a_6)</th>
<th>(a_7)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.3</td>
<td>-22.1</td>
<td>1078</td>
<td>1.82</td>
<td>0.208</td>
<td>0</td>
<td>-0.707</td>
<td></td>
</tr>
</tbody>
</table>

3.3 LINEAR YAW/PLANE MODEL WITH 3DOF

Figure 3.2 shows the linear yaw/plane model with 3 DOF (hereafter called 3DOF-L). The center of gravity (CG) of the car body has longitudinal motion at velocity \(U\), lateral motion at velocity \(V\), and yawing motion at yaw rate \(\dot{\psi}_1\). The trailer has an articulation angle \(\psi_2\). As shown in Figure 3.2, the system is telescoped laterally, and each axle set is represented by one wheel. In this model, the aerodynamic forces, the tire’s rolling resistance, the tire’s self-aligning torque, and the rolling and pitching motions are ignored. To derive the vehicle model and make all linearization, the following assumptions have been made: (1) the car’s front wheel steering angle \(\delta_f\) is small; (2) the articulation angle \(\psi_2\) is small; (3) all products of variables are ignored; and (4) the lateral tire force is a linear function of the tire side-slip angle; (5) for the zero initial conditions:

\[ \dot{\psi}_2 = \dot{\psi}_1 - \dot{\psi}_r \]  

(3.6)
where $\psi_i$ is defined as trailer yaw rate

From Newton’s law of dynamics, the equations of motion for the car are

$$m_c \cdot (\dot{U} - V \cdot \dot{\Psi}_1) = -X_f \cdot \cos \delta_f - X_r + X_{\psi_2}$$ \hspace{1cm} (3.7)

$$m_c \cdot (\dot{V} + U \cdot \dot{\Psi}_1) = Y_f + Y_r + X_f \cdot \sin \delta_f - Y_{\psi_2}$$ \hspace{1cm} (3.8)

$$I_c \cdot \ddot{\Psi}_1 = a \cdot Y_f - b \cdot Y_r + a \cdot X_f \cdot \sin \delta_f + d_1 \cdot Y_{\psi_2}$$ \hspace{1cm} (3.9)

and the equations of motion for the trailer are

$$m_t \cdot (\dot{U}_t - V_t \cdot \dot{\Psi}_t) = -X_t - Y_{\psi_2} \cdot \sin \Psi_2 - X_{\psi_2} \cdot \cos \Psi_2$$ \hspace{1cm} (3.10)

Fig.3.2. Schematic representation of the yaw plane model
\[ m_t \cdot (\dot{V}_t + U_t \cdot \dot{\Psi}_t) = Y_2 + Y_{\Psi 2} \cdot \cos \Psi_2 - X_{\Psi 2} \cdot \sin \Psi_2 \]  

\[ I_t \cdot \ddot{\Psi}_t = c \cdot Y_f - d_2 \cdot (X_{\Psi 2} \cdot \sin \Psi_2 - Y_{\Psi 2} \cdot \cos \Psi_2) \]  

(3.11)  

(3.12)

where \( r_f, r_r \), and \( r_t \) are the lateral force on the car front tire, the lateral force on the car rear tire, and the lateral force on the trailer tire, respectively. The linear tire model is expressed by the following equations

\[ Y_f = C_f \cdot \alpha_f \]  

(3.13)

\[ Y_r = C_r \cdot \alpha_r \]  

(3.14)

\[ Y_t = C_t \cdot \alpha_t \]  

(3.15)

where \( C_f, C_r \), and \( C_t \) are the tire cornering stiffness, \( \alpha_f, \alpha_r \), and \( \alpha_t \) are the side-slip angles of the tires. In this thesis, the tire cornering stiffness chooses the curve fitted parameter \( B \cdot C \cdot D \) of the Magic Formula tire model. The side-slip angles are given by

\[ \alpha_f = \frac{V + a \cdot \dot{\Psi}_1}{U} - \delta_f \]  

(3.16)

\[ \alpha_r = \frac{V - b \cdot \dot{\Psi}_1}{U} \]  

(3.17)

\[ \alpha_t = \dot{\Psi}_2 \cdot \frac{V - (d_1 + d_2 + c) \cdot \dot{\Psi}_1 + (d_2 + c) \cdot \dot{\Psi}_2}{U} \]  

(3.18)

The velocities of the articulation joint described in either the car-body fixed coordinate system or the trailer-body fixed coordinate system should be compatible, eliminating the
coupling reaction forces at the articulation joint, \( x_{v2} \) and \( y_{v2} \), from Eq. (3.7) through (3.12) leads to the linear yaw/plane model with 3 DOF expressed in the state space as

\[
M \{ \dot{X} \} + D \{ X \} + F \delta_f = 0
\] (3.19)

where the \( \delta_f \) is the car front wheel steering angle and the state variable vector is defined as

\[
\{ X \} = \begin{bmatrix} V & \Psi_1 & \Psi_2 \end{bmatrix}^T
\] (3.20)

The matrices \( M, D \) and \( F \) are listed in Appendix A. Table 3.2 list the notations and the primary parameters of the car-trailer system. This 3ODF-L model has been described by Sun[68].

<table>
<thead>
<tr>
<th>Car mass ( m_c )</th>
<th>2000 kg</th>
</tr>
</thead>
<tbody>
<tr>
<td>Car yaw inertia ( I_c )</td>
<td>3000 kgm²</td>
</tr>
<tr>
<td>Car dimension ( a )</td>
<td>1.5 m</td>
</tr>
<tr>
<td>Car dimension ( b )</td>
<td>1.7 m</td>
</tr>
<tr>
<td>Car dimension ( d_1 )</td>
<td>2.8 m</td>
</tr>
<tr>
<td>Trailer mass ( m_t )</td>
<td>2300 kg</td>
</tr>
<tr>
<td>Trailer yaw inertia ( I_t )</td>
<td>3900 kgm²</td>
</tr>
<tr>
<td>Trailer dimension ( c )</td>
<td>5 m</td>
</tr>
<tr>
<td>Trailer dimension ( d_2 )</td>
<td>0 m</td>
</tr>
<tr>
<td>Front tire cornering stiffness (combined) ( C_f )</td>
<td>-80000 Nm/rad</td>
</tr>
<tr>
<td>Rear tire cornering stiffness (combined) ( C_r )</td>
<td>-80000 Nm/rad</td>
</tr>
<tr>
<td>Trailer tire cornering stiffness (combined) ( C_t )</td>
<td>-60000 Nm/rad</td>
</tr>
</tbody>
</table>
3.4 NONLINEAR YAW/PLANE MODEL WITH 4DOF

For the nonlinear yaw/plane model with 4 DOF (hereafter called 4DOF-NL), the motions considered are the same as those for the 3DOF-L model as shown in Figure 3.2. The 4DOF-NL model incorporates some effects that are not taken into account in the 3DOF-L model. The tire model used in the 4DOF-NL model is based on the Magic Formula [46]. The tire forces in the 4DOF-NL model are derived without considering the lateral load transfer due to lateral accelerations. Moreover, aerodynamic lift and drag on the car and the trailer bodies are ignored in the nonlinear yaw/plane model. The derivation of the governing equations of motion for the 4DOF-NL model is conducted using Lagrange’s equations. The reader is referred to [66] for the detailed procedure for generating the governing equations of the nonlinear vehicle model.

As the most suitable way of solving the dynamical problem, Lagrange’s equations have been widely used to derive the equations of motion for the CT system. Lagrange’s equations of motion are derived by Hamilton’s Principle and the law of conservation of energy. In this approach, the dynamic system can be analyzed as a whole multibody system with \( n \) generalized coordinates, where \( n \) is the number of degrees of freedom that is equal to the number of equations of motion. Eq. (3.21) shows the Lagrange equations for a dynamic system.

\[
\frac{d}{dt} \left( \frac{\partial T_L}{\partial \dot{q}_i} \right) - \frac{\partial T_L}{\partial q_i} + \frac{\partial U_L}{\partial q_i} = Q_i \tag{3.21}
\]
where \( q \) is a generalized coordinate; \( T_L \) is the kinetic energy of the system; \( U_L \) is the potential energy of the system; and \( Q_q \) is a generalized force.

After the mathematic manipulation of Eq. (3.21), Lagrange equations become:

\[
F(\ddot{q}, \dot{q}, q, t) = Q_q
\]

Then, collect all elements about \( \ddot{q} \) and put them on the left side of the equation, other elements are put on the right side. The nonlinear equations can be shown as:

\[
[A(q, t)](\ddot{q}) = \{f(q, \dot{q}, t)\} + Q_q
\] (3.23)

In this nonlinear yaw/plane model of the CT system, the generalized coordinate can be listed as follows,

\[
q = [\int U dt \quad \int V dt \quad \Psi_1 \quad \Psi_2]^T
\] (3.24)

The governing equations of motion for this vehicle model are offered in Appendix B.

### 3.5 NONLINEAR YAW/PLANE MODEL WITH 6DOF

The nonlinear yaw/roll model with 6 DOF (hereafter called 6 DOF-NL) is generated using the procedure proposed [46]. For the 6DOF-NL model, the motions considered are the same as those used for the 4DOF-NL with the addition of roll motions, where \( \phi_1 \) is the roll angle of the car body and \( \phi_2 \) is the roll angle of the trailer body. Similar to the 4DOF-NL model, aerodynamic forces are ignored in the 6DOF-NL model; the Magic Formula is introduced to model the tire/road forces. Different with the 4DOF-NL model,
all tires are considered individually in the 6DOF-NL model, including the effects of
individual tire normal loads on the tire forces.

In order to account for the roll motions of the car-body and the trailer-body, the roll
model shown in Figure 3.3 is introduced in the 6DOF-NL vehicle model. Figure 3.3
shows the body, either the car-body or trailer-body, rolling around a longitudinal axis
with an angle $\phi$. In Figure 3.3, $m_1$ denotes the car unsprung mass, $m_2$ is the car sprung
mass, $T$ is the track between the left and right wheels, $S$ is the distance between the right
and left suspensions, $h_R$ is the vertical distance from the roll center to the road surface,
and $h\cos \phi$ is the vertical distance between the body CG to the roll center. With no other
lateral motion of the CG, the rotation around an axis passing through the body CG results
in a lateral motion $h\sin \phi$ of the axle. It is assumed that each axle moves laterally with
respect to the body CG and the lateral motion are calculated using the roll center shown
in Figure 3.3. The actual axle motion is determined by the motion of the whole vehicle
which is governed by the forces applied on the system.
Similar to the 4DOF-NL model, the governing equations of motion for the 6DOF-NL model is generated using Lagrange’s equations. The formulation of the governing equations is implemented using the method proposed by Anderson [66]. In this nonlinear CT model with 6-DOF, according to the calculation using the Lagrange’s equations, the nonlinear equations can be shown in Eq. (3.23), and the generalized coordinate can be shown as follows

\[ q = \left[ \int U dt \quad \int V dt \quad \Psi \quad \phi_1 \quad \phi_2 \right]^T \]  

(3.25)

### 3.6 NONLINEAR CarSim MODEL WITH 21 DOF

In the current research, a multibody system model based on the commercial software CarSim is generated to represent the car-trailer system [64]. For the towing unit, i.e., the
car, the following motions are considered: the car-body is treated as a rigid body with 6 DOF; each axle is modeled as a rigid body with 2 DOF (yaw and longitudinal translation); and each car wheel has one spin DOF. Similarly, regarding the trailing unit, the motions considered are: the trailer-body is treated as a rigid body with 6 DOF; the trailer axle is modeled as a rigid body with 2 DOF (yaw and longitudinal translation); and each trailer wheel has one spin DOF. The hitch connection between the car and the trailer results in the CarSim model with 21 DOF. The Magic Formula is used to model the tire/road forces. The geometrical parameters and configuration of the car model and trailer model are illustrated in Figure 3.4 and Figure 3.5.

![Figure 3.4 Geometrical parameters and configuration of the car model.](image-url)
3.7 SIMULATION RESULTS AND DISCUSSION

In the current research, the numerical simulations for the 3DOF-L, 4DOF-NL, and 6DOF-NL models are implemented in Matlab, while the CarSim model is running in CarSim software. The purpose of this section is to compare and evaluate the models and to analyze the difference between the nonlinear and linearized equations of motion for the CT system. In order to make the simulation results obtained from different models comparable, the aerodynamic effects in the nonlinear models are removed. At the same time, to make the simulation results more concise, all simulation data’s names are...
simplified as follows: the case of 3DOF-L model without controller is denoted as “n3”, the case of 4DOF-NL model without the controller is represented as “n4”, the case of 6DOF-NL model without controller is called “n6” and the case of CarSim model without a controller is named “nc”.

To compare and evaluate these models, the following case studies are conducted: 1) an eigenvalue analysis is performed to predict the unstable motion modes of the car-trailer system using the 3DOF-L model and the result is evaluated using the nonlinear models; 2) an emergency evasive test maneuver at low and high speed is emulated; 3) a single lane-change test maneuver at high speed is simulated.

### 3.7.1 Unstable Motion Mode Identification Based on Eigenvalue Analysis

In order to identify the unstable motion modes and predict the critical speed(s) of the CT system, an eigenvalue analysis is conducted using the 3DOF-L model. Note that the critical speed is a vehicle forward speed above which the vehicle will lose its stability. Then, the identified critical speed using the linear model will be evaluated in the time domain using the nonlinear models.

For the 3DOF-L model, the system matrix $A$ can be obtained from Eq. (3.26).

\[
A = -M^{-1}D
\]  (3.26)
with the system matrix \( A \), the eigenvalue vector analysis of the model can be conducted. One pair of complex eigenvalue \( S_{1,2} \) of the matrix may take the follow form Eq. (3.27).

\[
S_{1,2} = R_e + j \omega_d
\]  

(3.27)

where \( R_e \) and \( j \omega_d \) are the real and imaginary parts of the eigenvalue. The corresponding damping ratio is defined as,

\[
\zeta = -\frac{R_e}{\sqrt{R_e^2 + \omega_d^2}}
\]  

(3.28)

The damping ratio is a function of the vehicle forward speed. Figure 3.6 shows the relationship among the damping ratios (for two motion modes) and the vehicle forward speed. Curves 1 and 2 represent the damping ratios for motion modes 1 and 2, respectively. Curve 1 has the value close to 1.0, implying that the motion is well-damped and the corresponding motion mode is highly stable. For motion mode 2, once the forward speed is larger than 10.38 m/s, the damping ratio decreases with the speed and the ratio value approaches 0 when the speed is approximately 45 m/s. It is indicated that the critical speed is 45 m/s, above which the CT system will be liable to an unstable motion mode.
Figure 3.6. Damping ratios versus vehicle forward speed.

To validate the critical speed based on the 3DOF-L model and identify the unstable motion mode, the dynamic responses in the time domain are examined. To excite this unstable motion mode around the critical speed (45 m/s), the car’s front wheel steering angle input as shown in Figure 3.7 is used.
Figure 3.7. Car front wheel steering input for identifying the unstable motion mode

The simulations based on the CarSim model show that the critical speed is approximately 45.0 m/s and the corresponding unstable motion mode is trailer swaying and the CT system has skidded off the road. Figure 3.8 illustrates the unstable motion mode.

Figure 3.8. Visual representation of the trailer swaying and track-off road motions mode at 45 m/s
Figures 3.9-3.10 illustrate simulation results in terms of the time history of the car body’s trajectory and the trailer body’s trajectory, respectively, using the 3DOF-L, 4DOF-NL, 6DOF-NL, and the CarSim model at U=45 m/s.

The simulation results based on the 3DOF-L model show: the car and trailer bodies will lose stability, and the unstable motion mode is trailer swaying and track-off the road surface (the peak value of the road lateral distance is 3.5m, which is reserved by CarSim software) as shown in Figures 3.9-3.10. At the same speed, the simulation results show that the 4DOF-NL, 6DOF-NL, and the CarSim models agree with the 3DOF-L model in terms of the car body’s trajectory and the trailer body’s trajectory. According to the visual representation in Figure 3.8, the corresponding unstable motion mode of these three cases is trailer swaying and can lead the CT system to track-off the road.

Figure 3.9. Car body’s trajectory under the emergency evasive maneuver at U=45 m/s
The above analysis demonstrates that the simulation results in the time domain based on the 4DOF-NL, 6DOF-NL, and the CarSim models match those based on the 3DOF-L model. For the linear controller design, the above eigenvalue analysis method can also be used for critical speed(s) and unstable motion model(s) identification.

### 3.7.2 An Emergency Evasive Test Maneuver at Low and High Speeds

According to the “Driving the Speed Limit” in Ontario, Canada [65], typical speed limits are: 50–80 km/h (31–50 mph) on major arterial roads in urban and suburban areas, and 80–110 km/h (50–68 mph) on grade-separated expressways/freeways. Hence, to examine the dynamic behaviors of the models in typical evasive maneuvers at low and high speed...
maneuvers, the car front wheel steering angle input shown in Figure 3.7 is used to simulate an emergency evasive maneuver at the forward speed of 16.67 m/s (60km/h) and 27.79 m/s (100km/s).

3.7.2.1 An Emergency Evasive Test Maneuver at Low Speed

Figures 3.11-3.19 illustrate simulation results under the low speed maneuver in terms of the time history of the car lateral acceleration, trailer lateral acceleration, articulation angle, car yaw rate, trailer yaw rate, car roll angle, trailer roll angle, car body’s trajectory and trailer body’s trajectory, respectively, using the 3DOF-L, 4DOF-NL, 6DOF-NL, and the CarSim models. Under the emergency evasive maneuver at the forward speed of 16.67 m/s, the dynamic responses for all the four models are in good agreement. As shown in Figures 3.11 and 3.12 for all the cases, the car peak lateral accelerations are approximately 0.16 g and the trailer peak lateral accelerations are approximately 0.2 g. A close observation of the results shown in Figures 3.11-3.17 disclose that for all the dynamic responses, the CarSim model achieves the least peak values and the shortest settling time, while the 3DOF-L obtains the largest peak values and longest settling time. Moreover, the results shown in Figure 3.18 and 3.19 show that car body’s trajectory and trailer body’s trajectory for all the four models are in good agreement. Thus, the difference of linear and nonlinear tire models can be neglected at low lateral accelerations.
Figure 3.11. Car lateral acceleration versus time under the emergency evasive maneuver at $U=16.67$ m/s

Figure 3.12. Trailer lateral acceleration versus time under the emergency evasive maneuver at $U=16.67$ m/s
Figure 3.13. Articulation angle versus time under the emergency evasive maneuver at \( U=16.67 \) m/s

Figure 3.14. Car yaw rate versus time under the emergency evasive maneuver at \( U=16.67 \) m/s
Figure 3.15. Trailer yaw rate versus time under the emergency evasive maneuver at \(U=16.67\) m/s

Figure 3.16. Car roll angle versus time under the emergency evasive maneuver at \(U=16.67\) m/s
Figure 3.17. Trailer roll angle versus time under the emergency evasive maneuver at $U=16.67$ m/s

Figure 3.18. Car body’s trajectory under the emergency evasive maneuver at $U=16.67$ m/s
Figure 3.19. Trailer body’s trajectory under the emergency evasive maneuver at $U=16.67$ m/s

### 3.7.2.2 An Emergency Evasive Test Maneuver at High Speed

Figures 3.20-3.28 illustrate simulation results under the high speed maneuver in terms of the time history of the car lateral acceleration, trailer lateral acceleration, articulation angle, car yaw rate, trailer yaw rate, car roll angle, trailer roll angle, car body’s trajectory and trailer body’s trajectory, respectively, using the 3DOF-L, 4DOF-NL, 6DOF-NL, and the CarSim models. Under the single lane-change maneuver at the forward speed of 27.78 m/s, except the case of car body’s trajectory and trailer body’s trajectory, the dynamic responses for all the four models exhibit oscillations and they are damped out after a period of time, and they also have a similar variation trend. A close observation of the results shown in Figures 3.27-3.28 disclose that all models leave the path and exhibit the trailer swing motion except the 3DOF-L model.
Figure 3.20. Car lateral acceleration versus time under the emergency evasive maneuver at $U=27.78$ m/s

Figure 3.21. Trailer lateral acceleration versus time under the emergency evasive maneuver at $U=27.78$ m/s
Figure 3.22. Articulation angle versus time under the emergency evasive maneuver at 
$U=27.78$ m/s

Figure 3.23. Car yaw rate versus time under the emergency evasive maneuver at $U=27.78$ m/s
Figure 3.24. Trailer yaw rate versus time under the emergency evasive maneuver at $U=27.78 \text{ m/s}$

Figure 3.25. Car roll angle versus time under the emergency evasive maneuver at $U=27.78 \text{ m/s}$
Figure 3.26. Trailer roll angle versus time under the emergency evasive maneuver at $U=27.78$ m/s

Figure 3.27. Car body’s trajectory under the emergency evasive maneuver at $U=27.78$ m/s
The main reason for the difference of the simulation results between the linear and nonlinear vehicle models may result from the different tire models used. Simulation results in Figures 3.29-3.37 support this reasoning by the fact that only the 3DOF-L model can follow the desired path since with the linear tire model, the lateral tire force is not saturated and can increase without limitation. Simulation results in Figures 3.29-3.37 also demonstrate this reasoning by the fact that the other three models can’t follow the desired path since with the Magic Formula tire model, the lateral tire force is saturated when the tire side-slip angle is large. As shown in the Figure.3.37, the lateral tire force based on the nonlinear tire model will be saturated with the value of approvingly 6.3 KN, but in the case of the linear tire model, the lateral tire force still increase with the tire side-slip angle. Thus, it can be concluded that the dynamic responses for all the four
models are affected by different tire models. Hence, at large tire side-slip angles the
dynamic responses based on the linear and nonlinear are not in agreement.

Figure 3.29. Lateral force of the car front wheel versus time under the emergency evasive maneuver at $U=27.78$ m/s
Figure 3.30. Lateral force of the car rear wheel versus time under the emergency evasive maneuver at $U=27.78 \text{ m/s}$

Figure 3.31. Lateral force of the trailer wheel versus time under the emergency evasive maneuver at $U=27.78\text{m/s}$
Figure 3.32. Tire side-slip angle of the car front wheel versus time under the emergency evasive maneuver at $U=27.78$ m/s

Figure 3.33. Tire side-slip of the car rear wheel versus time under the emergency evasive maneuver at $U=27.78$ m/s
Figure 3.34. Tire side-slip of the trailer wheel versus time under the emergency evasive maneuver at $U=27.78 \text{ m/s}$

Figure 3.35. Car front tire side-slip angle vs. lateral force under the emergency evasive maneuver at $U=27.78 \text{ m/s}$
Figure 3.36. Car rear tire side-slip angle vs. lateral force under the emergency evasive maneuver at \( U = 27.78 \) m/s

Figure 3.37. Trailer tire side-slip angle vs. lateral force under the emergency evasive maneuver at \( U = 27.78 \) m/s
3.7.3 Single Lane-Change Test Maneuver at High Speed

To compare the dynamic characteristics of the nonlinear tire model and the linear tire model, and analyze the effects of the tire dynamic features on the CT systems by different tire models, a special single lane-change test maneuver is designed at the high speed (27.78m/s=100km/h) with the wheel steering angle input as shown in Figure 3.38.

![Figure 3.38. The car front wheel steering angle input under the single lane change](image)

Figures 3.39-3.47 provide simulation results in terms of the time history of the car lateral acceleration, trailer lateral acceleration, articulation angle, car yaw rate, trailer yaw rate, car roll angle, trailer roll angle, car body’s trajectory and trailer body’s trajectory, respectively, using the 3DOF-L, 4DOF-NL, 6DOF-NL, and the CarSim model at
U=27.78 m/s. As shown in Figures 3.39, 3.40, 3.42 and 3.43, in the case of the 3DOF-L, the oscillations of car lateral acceleration, trailer lateral acceleration, car yaw rate and trailer yaw rate, respectively, are damped out as time goes. It seems that the CT system represented by the 3DOF-L model can complete the test maneuver without losing stability. However, in reality, this is not the case. As illustrated in Figures 3.39 and 3.40, the peak lateral acceleration of the car and the trailer reach as high as 3.4 g and 4.2 g, respectively. Due to the limitation of the tire/road forces, the vehicle units’ lateral accelerations should not be larger than 1.0 g. Figures 3.46 and 3.47 show that the lateral displacement of the car and the trailer are approximately 80.0 m. This further clarifies the point that the dynamic responses of the 3DOF-L during the maneuver may not be true in practical operations. The reason for the unrealistic dynamic responses of the 3DOF-L may result from the tire lateral force saturation of the linear tire model.

At the same speed, the simulation results show that the 4DOF-NL, 6DOF-NL, and the CarSim models agree with one another in terms of the car lateral acceleration and trailer lateral acceleration. The car roll angle and trailer roll angle for the 6DOF-NL and the CarSim models are also in good agreement. On the other hand, as shown in Figures 3.41 and 3.42, for the cases of 4DOF-NL and 6DOF-NL, the articulation angle and car yaw rate increase with the time. Thus, the motions of the nonlinear models are not stable. Figures 3.46 and 3.47 indicate that the lateral displacements of the 4DOF-NL and 6DOF-NL increase with time. This further indicates that the motions of these models are not
stable. For the case of the CarSim model, as shown in Figures 3.46 and 3.47, this model also cannot complete the single lane change maneuver, neither.

A close observation of Figures 3.39 -3.47 reveals that the linear and nonlinear vehicle models have large differences in the simulation results. For example, in the case of the 3DOF-L model, the peak car lateral acceleration and the peak trailer lateral acceleration are as high as 3.4 g and 4.3 g, respectively, but in the case of nonlinear model, the peak later accelerations of the car and the trailer are approximately 0.8 g and 0.6 g. At the same time, the car roll angle and trailer roll angle will maintains constant values after approximately 2.5 deg and 1.2 deg.

The main reason for the huge difference of the simulation results between the linear and nonlinear vehicle models may result from the different tire models used. As expressed in Eqs. (3.13) through (3.15), for the linear tire model, the lateral tire force is a linear function of the tire side-slip angle. The lateral tire force increases with the tire side-slip angle. However, the Magic Formula specifies that once the tire side-slip angle reaches a certain value, the lateral tire force will be saturated.
Figure 3.39. Car lateral acceleration versus time under the single lane change maneuver at $U=27.78$ m/s

Figure 3.40. Trailer lateral acceleration versus time under the single lane change maneuver at $U=27.78$ m/s
Figure 3.41. Articulation angle versus time under the single lane change maneuver at U=27.78 m/s

Figure 3.42. Car yaw rate versus time under the single lane change maneuver at U=27.78 m/s
Figure 3.43. Trailer yaw rate versus time under the single lane change maneuver at $U=27.78$ m/s

Figure 3.44. Car roll angle versus time under the single lane change maneuver at $U=27.78$ m/s

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Figure 3.45. Trailer roll angle versus time under the single lane change maneuver at $U=27.78$ m/s

Figure 3.46. Car body’s trajectory under the single lane change maneuver at $U=27.78$ m/s
In this thesis, in order to make the simulation results obtained from different tire models comparable, the two wheels of each axle set is represented by one wheel, the peak value of the tire slip angle is limited to 90 degrees and the load transform effects on all tire models are not considered. Figures 3.48-3.53 illustrate simulation results in terms of the time history of lateral force of car front wheel, lateral force of car rear wheel, lateral force of trailer wheel, slip angle of car front wheel, slip angle of car rear wheel, slip angle of trailer wheel, respectively. As shown in Figures 3.51 and 3.53, unlike the 3DOF-L, in the cases of the nonlinear models, the tire side-slip angles increase with time, e.g., at the time of 12 second, the trailer tire side-slip angle for 6DOF-NL model is about 132 degrees. Thus, under the simulated maneuver, the nonlinear tire model works in its nonlinear...
range and the lateral tire forces are saturated. This reasoning is supported by the simulation results shown in Figures 3.48, 3.49 and 3.50 illustrating that for the nonlinear models, after a time instant, the lateral forces of the car front tire, car rear tire and trailer tire maintain about constant. However, for the 3DOF-L model, the corresponding tire lateral force oscillates.

Figure 3.48. Lateral force of the car front wheel versus time under the single lane change maneuver at \( U=27.78 \) m/s
Figure 3.49. Lateral force of the car rear wheel versus time under the single lane change maneuver at \( U=27.78 \text{ m/s} \).

Figure 3.50. Lateral force of the trailer wheel versus time under the single lane change maneuver at \( U=27.78 \text{ m/s} \).
Figure 3.51. Tire side-slip angle of the car front wheel versus time under the single lane change maneuver at $U=27.78 \text{ m/s}$

Figure 3.52. Tire side-slip angle of the car rear wheel versus time under the single lane change maneuver at $U=27.78 \text{ m/s}$
Figures 3.53-3.56 show the relationship between the tire lateral tire force and the tire side-slip angle based on the linear and nonlinear tire models. As the simulation results shown, in the case of the 3DOF-L model, the lateral force of each tire increases with the side-slip angle of the corresponding tire. However, in the case of the 4DOF-NL, 6DOF-NL, and the CarSim models, the simulation results based on the nonlinear tire model are in good agreement: the tire side-slip angle increase with time, but the lateral tire force will be saturated once the tire side-slip angle reaches a certain value. For the car front wheel, the lateral force will be saturated at approximately 4 KN. The lateral force of car rear wheel will be saturated at approximately 3 KN. For the trailer wheel, the saturation value is approximately 7 KN.
Figure 3.54. Car front tire side-slip angle vs. lateral force under the single lane change maneuver at U=27.78 m/s

Figure 3.55. Car rear tire side-slip angle vs. lateral force under the single lane change maneuver at U=27.78 m/s
3.8 SUMMARY

In this chapter, three car-trailer system models are compared against a multibody system model with 21 DOF developed in CarSim commercial software in terms of fidelity, complexity, and applicability for lateral motion controller design. Based on the comparative study of the simulation results of all four models, some insightful findings are summarized as follows:

(1) The linear model with 3 DOF is effective to predict the lateral stability (critical speed and unstable motion mode) of CT systems.
(2) Under the regular evasive maneuver at low lateral acceleration of the CT system (less than 0.5g, this value based on [71]), this linear model can be used to provide dynamic responses that are in good agreement with the nonlinear vehicle models. Thus, this linear yaw/plane model can be efficiently used for the lateral motion controller design under low lateral acceleration maneuver without considering the roll motions of the car and trailer.

(3) Under the regular single lane change maneuver at high lateral acceleration of the CT system (larger than 0.5g), this linear model can’t be used to provide dynamic responses that are in good agreement with the nonlinear vehicle models, because the lateral tire force saturation is not taken into account. Thus, this linear yaw/plane model is not suitable for the lateral stability controller design in this situation.

(4) Under the regular evasive maneuver at both low lateral acceleration of the CT system (less than 0.5g), and high lateral acceleration of the CT system (more than 0.5g), all the dynamic responses for the 4DOF-NL, 6DOF-NL and CarSim models well agree each other. Thus, it is demonstrated that compared with multibody system model with 21 DOF, the nonlinear yaw/role model, the 6 DOF is effective in terms of fidelity, complexity, and computational efficiency. The 3 dimensional model with 6 DOF can be used for lateral stability controller design considering maneuver at any lateral acceleration and accounting for roll motions of the car and the trailer. The controller performances derived from the nonlinear models are being examined in the next chapter.
Chapter 4

ATDB CONTROLLER DESIGN AND EVALUATION

4.1 INTRODUCTION

In this chapter, an active trailer differential braking (ATDB) controller is designed and evaluated to improve the lateral stability of car-trailer (CT) systems. Based on the 3-DOF linear model, the ATDB controller is derived using the Linear Quadratic Regular (LQR) technique, and then the resulting ATDB controller is examined using the nonlinear 4-DOF, the nonlinear 6-DOF and the 21-DOF CT models. A single lane-change maneuver has been simulated to evaluate the performance of the controller and the numerical results are compared with those of the baseline design. In comparison to a conventional CT safety control strategy, the new ATDB controller can be used for the nonlinear CT system model and improve the lateral stability of the CT system.

4.2 ATDB CONTROLLER DESIGN

In this section, the ATDB controller is designed to improve the lateral stability of CT systems. The proposed ATDB strategy is shown in Fig.4.1. The essential concept of this control strategy is to use the trailer yaw moment resulting from the trailer differential
braking force to enhance the stability of the CT system. As shown in Fig. 4.1, four sensors are employed to collect the vehicle state variables, including the car lateral speed, yaw rate, articulation angle, and the time rate of the articulation angle. The sensor information will be sent to the controller for manipulating the trailer braking system to produce an external trailer yaw moment through differential braking control. Then, this yaw moment will be applied on the CT model. The resulting yaw moment will prevent the unstable motion modes, such as the jack-knifing, trailer swing, and rollover.

![Diagram of the proposed ATDB strategy

Figure.4.1 The proposed ATDB strategy

With the ATDB control strategy, the vehicle state variables from the seniors are analyzed and the performance measures are to be calculated. If there is a potential for an unstable motion models, such as the jack-knifing, the ATDB controller will manipulate the trailer braking system. Through the trailer differential braking, the resulting yaw moment of the trailer will align the trailer with the towing unit. Thus, the articulation angle between the
towing and trailer units can be reduced and jack-knifing can be prevented. In this process, the external trailer yaw moment $M_Z$ is produced through intelligently adjusting differential braking forces on the left or right-side wheels of the trailer, which can be defined as in Eq. (4.1)

$$M_Z = \frac{T}{2} \cdot \Delta F_t$$

where $\Delta F_t$ is the longitudinal force difference between the left and the right wheels of the trailer and $T$ is the track between the left and right wheels of the trailer.

### 4.3 THE ATDB CONTROLLER DESIGN 3-DOF LINEAR MODEL

In the case where the ATDB control system is involved, the CT system model is augmented by introducing an additional external yaw moment on the trailer to improve the lateral stability of CT systems. Hence, adding the yaw moment $M_Z$ on the right side of Eq. (3.17) and resulting in Eq. (4.2)

$$I_t \cdot \dot{\Psi}_t = c \cdot Y_f - d \cdot (X_{\Psi_2} \cdot \sin \Psi_2 - Y_{\Psi_2} \cdot \cos \Psi_2) + M_z$$

At the same time, the resulting state space form of Eq. (3.24) becomes Eq. (4.3),

$$M \dot{\hat{X}} + D \{X\} + C\delta u + F \delta f = 0$$

where the state variable vector $X$ and the matrices $M$, $D$ and $F$ are defined the same as the Eq. (3.24). The control variable vector is defined as
and the control matrix, $C_B$, is given in Appendix B.

In order to design the ATDB controller using LQR technique, the governing equation of motion of the 3-DOF model is expressed as

$$
\dot{X} = (A - B \cdot K) \cdot X
$$

where,

$$
A = -M^{-1} \cdot D
$$

$$
B = -M^{-1} \cdot C_B
$$

The LQR controller design can be described as an optimization problem, and the goal is to minimize the performance index:

$$
J = \int_0^\infty \left[ Q_1 \left( \dot{\mathbf{V}} + \mathbf{U} \cdot \Psi_1 \right)^2 + Q_2 \left( \dot{\mathbf{U}} + \mathbf{U} \cdot \Psi_1 \right)^2 + Q_3 M_x^2 \right] dt
$$

subject to Eq. (4.3). By solving the algebraic Riccati equation, the solution of the optimization problem is the control vector of the form:

$$
u = -K \cdot X = -(K_1 \cdot \mathbf{V} + K_2 \cdot \Psi_1 + K_3 \cdot \Psi_2 + K_4 \cdot \Psi_2)
$$
where $K$ is the control gain matrix, which can be determined by the LQR algorithm, and $X$ and $u$ are the state and control variable vectors defined by Eq. (4.3)-(4.4), respectively. In Eq. (4.8), $Q_1$, $Q_2$, and $Q_3$ are the weighting factors that impose penalties upon the magnitude and duration of the lateral acceleration at the car’s Center of Gravity (CG), the lateral acceleration at the trailer CG, and the active trailer yaw moment, $M_Z$. Note that the third term on the right hand side of Eq. (4.8) represents the energy consumption of the ATDB control system.

As shown in Eq. (4.8), by coordinating the relationship between the lateral accelerations at the car CG and the trailer CG, the articulation angle between the car and trailer and the roll angles of car and trailer bodies may be controlled. Thus, the dangerous jack-knifing and trailer swing may be prevented. Moreover, both the magnitudes of the lateral accelerations at the car CG and the trailer CG are to be minimized. Thus, the rollover stability will be enhanced. The above design considerations will be justified for other models.

### 4.4 THE ATDB CONTROLLER DESIGN FOR NONLINEAR MODELS

In the thesis, the forward speed $U$ is regarded as a constant. Therefore, in order to achieve the ATDB control strategy, the nonlinear equations of Eq. (3.23) can be expressed as nonlinear functions of both the state variable vector and time as,

$$M(\int X dt, t) \cdot \dot{X} = F(\int X dt, X, t)$$

(4.10)
where \( M(\int Xdt, t) \) is the generalized mass matrix, \( F(\int Xdt, X, t) \) is the generalized force, and they are nonlinear functions of both system’s the state variable vector \( X \) and the simulation time \( t \).

In the case of the nonlinear 4-DOF CT model, the essential concept of the ATDB control strategy is to introduce a stabilizing external yaw moment on the trailer in order to improve the lateral stability of the CT system. Hence, with the ATDB control system involved, the state space form expressed in Eq. (4.10) becomes the Eq. (4.11)

\[
M(\int Xdt, t) \cdot \dot{X} = F(\int Xdt, X, t) + C_B \cdot u
\]  

(4.11)

where \( X \) is shown as Eq. (3.20), \( u \) is shown as Eq. (4.9) by using the set control gains \( K \) which are based on the 3-DOF linear model and the LQR technique. \( C_B \) is the same as that for the 3-DOF model. The rollover stability will also be enhanced.

In the 6-DOF nonlinear model case, all considered motions are the same as those used for the 4DOF-NL with the addition of roll motions. Therefore, all dynamics analysis must include lateral motions and roll motions of CT system. In the following detailed design of the ATDB controller, these characteristics will be considered.

According to calculations by using the Lagrange equations, the nonlinear simulation equations of the form can be shown as Eq. (3.23). Then, in order to let the 6DOF-NL model adapt the ATDB control strategy, the Eq. (3.23) must be collated as nonlinear
functions of both state variable and time as shown as Eq. (4.10) by the above rules, where \( X \) is the state variable vector of the 6DOF-NL model and is defined as,

\[
X = [\dot{V}, \dot{\psi}_1, \dot{\psi}_2, \dot{\psi}_2, \dot{\phi}_1, \dot{\phi}_2]^T
\]  

(4.12)

However, not like the 4DOF-NL model, all six tires are considered individually in the 6DOF-NL model, including the effects of individual tire force. According to these dynamics features of the 6DOF-NL model, tire forces of each tire affects not only lateral motion but also roll motion. Hence, the essential concept of this case is that through intelligently adjusted differential braking forces on the left or right-side wheels of trailer, we can improve the lateral stability of CT systems. The object of the ATDB strategy is to manipulate the differential braking forces \( \Delta F_t \) on left and the right-side wheels of the trailer, which can be derived from Eq. (4.1),

\[
\Delta F_t = 2 \cdot \frac{M_z}{t}
\]

(4.13)

With the ATDB control system involved, the state space form of the governing equation becomes,

\[
M(\int X dt, t) \cdot \dot{X} = F(\int X dt, X, t) + F_B(\int X dt, X, \Delta F_t, t)
\]

(4.14)

where \( F_B(\int X dt, X, t) \) is defined as the generalized control force, which are nonlinear functions of the differential braking force \( \Delta F_t \) on the left and right-side wheels of the trailer, and the state vector \( X \) is defined as:

\[
X = [\dot{V}, \dot{\psi}_1, \dot{\psi}_2, \dot{\psi}_2, \dot{\phi}_1, \dot{\phi}_2]^T
\]
trailer, the state variable vector $X$ and the simulation time $t$. The nonlinear function can take the following form,

$$F_B(\int X dt, X, \Delta F_t, t) = 2 \cdot \Delta F_t \cdot C_{FB} = \frac{M_Z}{T} \cdot C_{FB} = \frac{4}{T} \cdot C_{FB} \cdot u$$  \hspace{1cm} (4.15)$$

where $M_Z$ is the yaw moment and can be produced by using the set control gains $K$ which are based on the 3-DOF linear model and the LQR technique. $C_{FB}$ is the generalized coefficient matrix of the ATDB control system and it is defined as:

$$C_{BF} := \begin{bmatrix}
\psi_2 \\
-\psi_2^2 P_k \phi_1 - \psi_2 d_1 \\
\psi_2^2 P_i - \psi_2 P_k \phi_2 + \psi_2 P_i + \psi_2^2 P_k \phi_2 \\
0 \\
-\psi_2 (H_H + H_{TC}) \\
- H_{TC} \psi_2
\end{bmatrix}$$  \hspace{1cm} (4.16)$$

where $H_H$ is the vertical distance from the car C.G to hitch, $H_{TC}$ is the vertical distances of the roll center of the trailer suspension, respectively. $P_i, P_j$ and $P_k$ are the longitudinal, lateral and vertical position of the center of trailer wheels, which are measured with respect to the hitch point of the trailer.
4.5 THE ATDB CONTROLLER DESIGN FOR NONLINEAR MODELS FOR 21-DOF NONLINEAR MODEL

To implement the ATDB control strategy, the ATDB controller is constructed in Matlab/Simulink and the 21-DOF nonlinear model is generated in the CarSim software; then the ATDB controller in combined with the CarSim model through the interface of the two software packages. With the CarSim model, the yaw moment is generated through the manipulation of individual trailer tire braking force. Fig.4.2 shows the working principle of the brake system of the CarSim model.

The brake line pressure (transport delayed master cylinder pressure) is modified to provide a pressure in the brake actuator. A tabular function relates the pressure to brake torque.

Figure.4.2 The working principle of the brake system of the CarSim model.

For this case, in order to achieve this ATDB control strategy, the master cylinder is controlled by the controller to provide different brake line pressure on the brake actuator of the corresponding wheel, then through a tabular function based on the principle of friction to produce different brake torques. Eventually, those brake torque are converted different brake force on the corresponding wheel to control the lateral and roll motions.
Hence, there are two differential braking control strategies: one to control the trailer (ATDB controller) and the other to control both the car (Active Car Differential Braking controller) and the trailer (ATDB controller). The Active Car Differential Braking (ACDB) controller is based on the yaw moment control strategy of the car body. Note that the ABS controller is disabled in this case.

### 4.6 NUMERICAL SIMULATION

The vehicle system models introduced in chapter 3, and the corresponding ATDB controllers described in this chapter are jointly constructed and integrated in Matlab©/Simulink© as shown in Fig. 4.3, 4.4, 4.5, and 4.6. To perform the numerical simulations the vehicle system parameters take the values listed in Table 1, other parameters take the values provided by the “E-Class, Sedan w/1A Trailer” model generated in the CarSim software [64]. The numerical simulations are conducted under the emulated single lane-change maneuver. For the purpose of comparison, the numerical simulations based on each CT model both with and without the ATDB controller have been performed. The respective numerical simulation results are presented and discussed in the following subsection.
Figure.4.3 Integration of the 3-DOF linear CT model with the ATDB controller.
Figure 4.4 Integration of the 4-DOF nonlinear CT model with the ATDB controller.
Figure 4.5 Integration of the 6-DOF nonlinear CT model with the ATDB controller.
Figure 4.6 Integration of the 21-DOF nonlinear CarSim model with the ATDB controller.
4.7 SIMULATION RESULTS AND DISCUSSION

To examine the proposed design method, which has been applied to the design of the CT system represented by the 3DOF-L, 4DOF-NL, 6DOF-NL and CarSim models as introduced in section 4.4-4.6, the numerical simulations based on the CT models with and without the ATBD controller will be performed. At the same time, to make the simulation results more concise, all simulation data’s names are simplified as follows: the case of the 3DOF-L model with the controller is denoted as “l3”, the 4DOF-NL model with the controller is represented as “l4”, the 6DOF-NL model with the controller is named “l6”, the CarSim model with the controller is replaced by “2c”, and the CarSim model with both the ATBD controller and the ACBD (Active Car Differential Braking) controller is denoted as “6c”.

This chapter chooses two testing maneuvers which have been investigated in the chapter 3: 1) an emergency evasive maneuver at low lateral acceleration of the CT system (less than 0.5 g); 2) a single lane-change test maneuver of high lateral acceleration of the CT system (larger than 0.5 g). This lateral acceleration value get from the relevant work [71].

4.7.1 An Emergency Evasive Maneuver at Low Lateral Acceleration

In this subsection, an emergency evasive maneuver at forward speed 27.78 m/s (100km/h) is emulated to examine the dynamic behaviors of all the four CT models with the corresponding ATBD controller at low lateral acceleration of CT system. The car front wheel steering angle input is shown in Figure 3.7.
Figure 4.7 offers the simulation results in terms of the car lateral acceleration versus time for the designs with and without the ATDB controller, respectively, using the 3DOF-L, 4DOF-NL, 6DOF-NL models. In the case of the 3DOF-L model with the ATDB controller, the maximum peak value of the car acceleration variation is 0.1 g, reducing by 77.8% from the baseline value of 0.45 g; the settling time of the car lateral acceleration oscillation is 5.5 seconds, decreasing by 6.5 seconds from the baseline value of 12 seconds. In the case of the 4DOF-NL model with the ATDB controller, the maximum peak value of the car acceleration variation is 0.1 g, reducing 78.2% from the baseline value of 0.46 g; the settling time of the car lateral acceleration oscillation is 5.5 seconds, decreasing 8.5 seconds from the baseline value of 14 seconds. In the case of the 6DOF-NL model with the ATDB controller, the maximum peak value of the car acceleration variation is 0.1 g, reducing 75% from the baseline value of 0.4 g; the settling time of the car lateral acceleration oscillation is 5.5 seconds, decreasing 8.5 seconds from the baseline value of 14 seconds. At a smaller car lateral acceleration, the car will have less chance to undergo the rollover. As shown in Figure 4.8, the maximum peak value of the car roll angle is 0.3 deg, reducing 76.9% from the baseline value of 1.3 deg.
Figure 4.7. Car lateral acceleration versus time with and without ATDB controller under the emergency evasive maneuver at $U=27.78$ m/s using 3DOF-L, 4DOF-NL and 6DOF-NL models

Figure 4.8. Car roll angle versus time with and without ATDB controller under the emergency evasive maneuver at $U=27.78$ m/s using 6DOF-NL model
Figure 4.9 shows the trailer lateral acceleration versus time for the case of the 3DOF-L, 4DOF-NL and 6DOF-NL models with and without the ATDB controller. Compared with the baseline design, the system with the ATDB controller has lower trailer lateral acceleration: in the 3DOF-L case with the ATDB controller, the maximum trailer acceleration peak value is 0.15 g, reducing 75% from the baseline value of 0.6 g and the settling time of the trailer lateral acceleration oscillation is 5 seconds, decreasing 9 seconds from the baseline value of 14 seconds; in the 4DOF-NL case with the ATDB controller, the maximum trailer acceleration peak value is 0.15 g, reducing 74.1% from the baseline value of 0.58 g and the settling time of the trailer lateral acceleration oscillation is 5 seconds, decreasing 13 seconds from the baseline value of 18 seconds; in the 6DOF-NL case with the ATDB controller, the maximum trailer acceleration peak value is 0.15 g, reducing 72.7% from the baseline value of 0.55 g and the settling time of the trailer lateral acceleration oscillation is 5 seconds, decreasing 7 seconds from the baseline value of 12 seconds. With a lower trailer lateral acceleration, the trailer will have less chance to undergo the rollover. This is demonstrated with the result shown in Figure 4.10. In the case of the 6DOF-NL with the ATDB controller, the maximum peak value of the roll angle is 0.25 deg, reducing 77.3% from the baseline value of 1.1 deg.
Figure 4.9. Trailer lateral acceleration versus time with and without the ATDB controller under the emergency evasive maneuver at $U=27.78$ m/s using the 3DOF-L, 4DOF-NL and 6DOF-NL models.

Figure 4.10. Trailer roll angle versus time with and without the ATDB controller under the emergency evasive maneuver at $U=27.78$ m/s using the 6DOF-NL model.
Figure 4.11 illustrates the time history diagrams of the articulation angle between the car and trailer in both designs with and without the ATDB controller, respectively, using the 3DOF-L, 4DOF-NL, 6DOF-NL models. In the 3DOF-L case with the ATDB controller, the maximum peak value of the articulation angle is 0.6 deg, reducing 90.1% from the baseline value of 6.1 deg; the settling time of the angle oscillation is 5 seconds, decreasing 7 seconds from the baseline value of 12 seconds. In the 4DOF-NL case with the ATDB controller, the maximum peak value of the articulation angle is 0.6 deg, reducing 98.6% from the baseline value of 6.5 deg; the settling time of the angle oscillation is 5 seconds, decreasing 10 seconds from the baseline value of 15 seconds. In the 6DOF-NL case with ATDB controller, the maximum peak value of the articulation angle is 0.6 deg, reducing 92.5% from the baseline value of 8 deg; the settling time of the angle oscillation is 5 seconds, decreasing 5 seconds from the baseline value of 10 seconds. The results shown in Figure 4.11 indicate that the ATDB controller can effectively reduce the articulation angle to improve the dynamic impact of jack-knifing.
Figure 4.11. Articulation angle versus time with and without the ATDB controller under the emergency evasive maneuver at \( U = 27.78 \text{ m/s} \) using the 3DOF-L, 4DOF-NL and 6DOF-NL models.

The time history diagrams of the car’s yaw rate of the 3DOF-L, 4DOF-NL and 6DOF-NL models for both the designs with and without the ATDB controller are shown in Figure 4.12. The maximum peak value of the oscillation for the design with the ATDB controller based on the 3DOF-L model is only 2.5 deg/s, decreasing 76.2% from the baseline value of 10.5 deg/s, and the car yaw rate oscillation is damped out after only 5 seconds, a reduction of 7 seconds from the corresponding baseline value of 12 seconds. The maximum peak value of the oscillation for the design with the ATDB controller based on the 4DOF-NL model is only 2.5 deg/s, decreasing 77.2% from the baseline value of 11 deg/s, and the car yaw rate oscillation is damped out after only 5 seconds, a reduction of 9 seconds from the corresponding baseline value of 14 seconds. The maximum peak value of the oscillation for the design with the ATDB controller based on
the 6DOF-NL model is only 2.5 deg/s, decreasing 73.7% from the baseline value of 9.5 deg/s, and the car yaw rate oscillation is damped out after only 5 seconds, a reduction of 4 seconds from the corresponding baseline value of 9 seconds. The results shown in Figure 4.12 indicate that the ATDB controller installed on the trailer has a significant dynamic impact on the leading unit.

Figure 4.12. Car yaw rate versus time with and without the ATDB controller under the emergency evasive maneuver at U=27.78 m/s using the 3DOF-L, 4DOF-NL and 6DOF-NL models.

The time history diagrams of the trailer yaw rate based on the 3DOF-L, 4DOF-NL and 6DOF-NL models for both designs with and without the ATDB controller are illustrated in Figure 4.13. In the design based on the 3DOF-L with the ATDB controller, the maximum peak value is 4 deg/s, reducing to 77.8% from the baseline value of 18 deg/s;
the settling time is 4.5 seconds, decreasing 8.5 seconds from the baseline value of 13 seconds. In the design based on the 4DOF-NL model with the ATDB controller, the maximum peak value is 4 deg/s, reducing to 78.4% from the baseline value of 18.5 deg/s; the settling time is 4.5 seconds, decreasing 6.5 seconds from the baseline value of 11 seconds. In the design with the ATB controller of the 6DOF-NL model, the maximum peak value is 4 deg/s, reducing to 77.1% from the baseline value of 17.5 deg/s; the settling time is 4.5 seconds, decreasing 11.5 seconds from the baseline value of 16 seconds. The ATDB controller directly contributes the improvement of the trailer yaw rate response and effectively reduces the influence by trailer swing.

Figure 4.13. Trailer yaw rate versus time with and without the ATDB controller under the emergency evasive maneuver at U=27.78 m/s using the 3DOF-L, 4DOF-NL and 6DOF-NL models
Figures 4.14 and 4.15 illustrate the car body’s trajectory and the trailer body’s trajectory based on the three models. In the design based on the 3DOF-L model with the ATDB controller, the maximum lateral displacement of the car body’s trajectory is 2.8m, reducing by 58.8% from the baseline value of 6.8m; the maximum lateral displacement of the trailer body’s trajectory is 2.8m, reducing by 62.1% from the baseline value of 7.4m. In the design based on the 4DOF-NL model with the ATDB controller, the maximum lateral displacement of the car body’s trajectory is 2.8m, reducing by 61.1% from the baseline value of 7.2m; and the maximum lateral displacement of the trailer body’s trajectory is 2.8m, reducing by 67.4% from the baseline value of 8.6m. In the design based on the 6DOF-NL model with the ATDB controller, the maximum lateral displacement of the car body’s trajectory is 2.8m, reducing to 60% from the baseline value of 7m; and the maximum lateral displacement of the trailer body’s trajectory is 2.8m, reducing to 65% from the baseline value of 8m. All simulation results show that the ATDB controller is effective to improve the lateral stability of the CT system under the emergency evasive maneuver at low lateral acceleration.
Figure 4.14. Car body’s trajectory versus time with and without the ATDB controller under the emergency evasive maneuver at U=27.78 m/s using the 3DOF-L, 4DOF-NL and 6DOF-NL models

Figure 4.15. Trailer body’s trajectory versus time with and without the ATDB controller under the emergency evasive maneuver at U=27.78 m/s using the 3DOF-L, 4DOF-NL and 6DOF-NL models
To quantitatively analyze the CT system lateral stability based on the cases of the designs with and without the ATDB controller, the dynamic responses of the 3DOF-L, 4DOF-NL and 6DOF-NL models in the emergency evasive maneuver are summarized in Tables 4.1-4.3.

Table 4.1 the dynamic responses of the 3DOF-L model with without the ATDB controller in the emergency evasive maneuver

<table>
<thead>
<tr>
<th>3DOF-L Low g (peak value)</th>
<th>Car lateral acceleration(g)</th>
<th>Trailer lateral acceleration(g)</th>
<th>Articulation angle (deg)</th>
<th>Car yaw rate(deg/s)</th>
<th>Trailer yaw rate(deg/s)</th>
<th>Car lateral displacement(m)</th>
<th>Trailer lateral displacement(m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>3DOF-L-n3</td>
<td>0.45</td>
<td>0.6</td>
<td>6.1</td>
<td>10.5</td>
<td>18</td>
<td>6.8</td>
<td>7.4</td>
</tr>
<tr>
<td>3DOF-L-13</td>
<td>0.1</td>
<td>0.15</td>
<td>0.6</td>
<td>2.5</td>
<td>4</td>
<td>2.8</td>
<td>2.8</td>
</tr>
<tr>
<td>Improvement</td>
<td>77.8%</td>
<td>75%</td>
<td>90.1%</td>
<td>76.2%</td>
<td>58.8%</td>
<td>62.1%</td>
<td></td>
</tr>
</tbody>
</table>

Table 4.2 the dynamic responses of the 4DOF-NL model with without the ATDB controller in the emergency evasive maneuver

<table>
<thead>
<tr>
<th>4DOF-NL Low g (peak value)</th>
<th>Car lateral acceleration(g)</th>
<th>Trailer lateral acceleration(g)</th>
<th>Articulation angle (deg)</th>
<th>Car yaw rate(deg/s)</th>
<th>Trailer yaw rate(deg/s)</th>
<th>Car lateral displacement(m)</th>
<th>Trailer lateral displacement(m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>4DOF-NL-n4</td>
<td>0.46</td>
<td>0.58</td>
<td>6.5</td>
<td>11</td>
<td>18.5</td>
<td>7.2</td>
<td>8.6</td>
</tr>
<tr>
<td>4DOF-NL-14</td>
<td>0.1</td>
<td>0.15</td>
<td>0.6</td>
<td>2.5</td>
<td>4</td>
<td>2.8</td>
<td>2.8</td>
</tr>
<tr>
<td>Improvement</td>
<td>78.2%</td>
<td>74.1%</td>
<td>98.6%</td>
<td>77.2%</td>
<td>61.1%</td>
<td>67.4%</td>
<td></td>
</tr>
</tbody>
</table>

Table 4.3 the dynamic responses of the 6DOF-NL model with without the ATDB controller in the emergency evasive maneuver

<table>
<thead>
<tr>
<th>6DOF-NL Low g (peak value)</th>
<th>Car lateral acceleration(g)</th>
<th>Trailer lateral acceleration(g)</th>
<th>Articulation angle (deg)</th>
<th>Car yaw rate(deg/s)</th>
<th>Trailer yaw rate(deg/s)</th>
<th>Car lateral displacement(m)</th>
<th>Trailer lateral displacement(m)</th>
<th>Car roll angle(deg)</th>
<th>Trailer roll angle(deg)</th>
</tr>
</thead>
<tbody>
<tr>
<td>6DOF-NL-n6</td>
<td>0.4</td>
<td>0.55</td>
<td>8</td>
<td>9.5</td>
<td>17.5</td>
<td>7</td>
<td>8</td>
<td>1.3</td>
<td>1.1</td>
</tr>
<tr>
<td>6DOF-NL-16</td>
<td>0.1</td>
<td>0.15</td>
<td>0.6</td>
<td>2.5</td>
<td>4</td>
<td>2.8</td>
<td>2.8</td>
<td>0.3</td>
<td>0.25</td>
</tr>
<tr>
<td>Improvement</td>
<td>75%</td>
<td>72.7%</td>
<td>92.5%</td>
<td>73.7%</td>
<td>60%</td>
<td>65%</td>
<td>76.6%</td>
<td>77.3%</td>
<td></td>
</tr>
</tbody>
</table>
Figure 4.16 offers the simulation results based on the CarSim model in terms of the car lateral acceleration versus time for the designs with the ATBD controller, with both the ATBD and the ACDB controllers, and the baseline vehicle without any controller. In the case of the ATDB controller denoted as “2c”, the maximum peak value of the car acceleration variation is 0.2 g, reducing 50% from the baseline value of 0.4 g; the settling time of the car lateral acceleration oscillation is 6 seconds, decreasing 4 seconds from the baseline value of 10 seconds. In the case of both the ATBD and the ACDB controllers, the maximum peak value of the car acceleration is 0.08 g, reducing 80% from the baseline value of 0.4 g; the settling time of the car lateral acceleration oscillation is 5 seconds, decreasing 5 seconds from the baseline value of 10 seconds. At a lower car lateral acceleration, the car with the controller will have less chance to undergo the rollover. As shown in Figure 4.17, in the case of the ATDB controller, the maximum peak value of the roll angle is 0.7 deg, reducing 36.4% from the baseline value of 1.1 deg; and the maximum peak value of the roll angle is 0.25 deg reducing 77.3% from the baseline value of 1.1 deg in the case of the ATDB and the ACDB controllers.
Figure 4.16. Car lateral acceleration versus time under the CarSim model with and without the controller under the emergency evasive maneuver at U=27.78 m/s

Figure 4.17. Car roll angle versus time under the CarSim model with and without the controller under the emergency evasive maneuver at U=27.78 m/s
The trailer lateral acceleration versus time based on the designs using the CarSim model with the ATBD controller, both the ATBD and the ACDB controllers, and the baseline vehicle is illustrated in Figure. 4.18. Compared with the baseline design, the system with the ATDB controller has a better trailer lateral acceleration response: in the case of ATDB controller, the maximum trailer acceleration peak value is 0.28 g, reducing 41.7% from the baseline value of 0.48 g and the settling time of the trailer lateral acceleration oscillation is 6 seconds, decreasing 7 seconds from the baseline value of 13 seconds; in case of the ATDB and the ACDB controllers, the maximum trailer acceleration peak value is 0.1 g, reducing 79.2% from the baseline value of 0.48 g and the settling time of the trailer lateral acceleration oscillation is 5.5 seconds, decreasing 7.5 seconds from the baseline value of 13 seconds. With a lower trailer lateral acceleration, the trailer with the controller will have less chance to undergo the rollover. As shown in Figure 4.19, the maximum peak value of the roll angle is 0.49 deg, reducing 38.8% from the baseline value of 0.8 deg in the case of the ATDB controller; and the maximum peak value of the roll angle variation is 0.08 deg reducing 90% from the baseline value of 0.8 deg in the case of the ATDB and the ACDB controllers.
Figure 4.18. Trailer lateral acceleration versus time under the CarSim model with and without the controller under the emergency evasive maneuver at $U=27.78$ m/s.

Figure 4.19. Trailer roll angle versus time under the CarSim model with and without the controller under the emergency evasive maneuver at $U=27.78$ m/s.
Figure 4.20 illustrates the time history diagrams of the articulation angle between the car and trailer in the designs with the ATBD controller, with both the ATBD and the ACDB controllers, and the baseline vehicle without any controller based on the CarSim models. In the case with the ATDB controller, the maximum peak value of the articulation angle is 2 deg, reducing 71.4% from the baseline value of 7 deg; the settling time of the angle oscillation is 5.5 seconds, decreasing by 6.5 seconds from the baseline value of 12 seconds. In the case with both the ATDB and the ACDB controllers, the maximum peak value of the articulation angle is 1 deg, reducing to 85.7% from the baseline value of 7 deg; the settling time of the angle oscillation is 5 seconds, decreasing 7 seconds from the baseline value of 12 seconds. The results shown in Figures 4.20 indicate that the ATDB and ACDB controllers can effectively reduce the articulation angle to improve the dynamic impact of jack-knifing.

Figure 4.20. Articulation angle versus time under the CarSim model with and without the controller under the emergency evasive maneuver at U=27.78 m/s
The time history diagrams of the car’s yaw rate of the CarSim models for the designs with the ATBD controller, both the ATBD and the ACDB controllers, and the baseline vehicle are shown in Figure. 4.21. The maximum peak value of the oscillation for the design with the ATDB controller is only 5 deg/s, decreasing 37.5% from the baseline value of 8 deg/s, and the car yaw rate oscillation is damped out after only 6 seconds, a reduction of 4 seconds from the corresponding baseline value of 10 seconds. The maximum peak value of the oscillation for the design with both the ATDB and the ACDB controllers is only 1 deg/s, decreasing 87.5% from the baseline value of 8 deg/s, and the car yaw rate oscillation is damped out after only 5 seconds, a reduction of 5 seconds from the corresponding baseline value of 10 seconds. The results shown in Figure.4.21 indicate that the ATDB and the ACDB controllers installed on the trailer and the car have a significant dynamic impact on the leading unit.

![Car yaw rate versus time](image)

Figure 4.21. Car yaw rate versus time of the CarSim model with and without the controller under the emergency evasive maneuver at U=27.78 m/s
The time history diagrams of the trailer yaw rate based on the CarSim models for the designs with the ATBD controller, both the ATBD and the ACDB controllers, and the baseline vehicle are illustrated in Figure 4.22. In the design with the ATDB controller, the maximum peak value is 2 deg/s, reducing to 86.7% from the baseline value of 15 deg/s; the settling time is 4.5 seconds, decreasing to 7.5 seconds from the baseline value of 12 seconds. In the design with both ATDB and ACDB controllers, the maximum peak value is 7.5 deg/s, reducing to 50% from the baseline value of 15 deg/s; the settling time is 4.5 seconds, decreasing 7.5 seconds from the baseline value of 12 seconds. The ATDB and the ACDB controllers directly contribute the improvement of the trailer yaw rate response and effectively reduce the influence by trailer swing.

Figure 4.22. Trailer yaw rate versus time of the CarSim model with and without the controller under the emergency evasive maneuver at U=27.78 m/s
Figures 4.23 and 4.24 illustrate the car body’s trajectory and the trailer body’s trajectory for the CarSim model. In the design with the ATDB controller, the maximum lateral displacement of the car body’s trajectory is 5.2 m, reducing 17.3% from the baseline value of 6.1 m; and the maximum lateral displacement of the trailer body’s trajectory is 5.2 m, reducing to 15.4% from the baseline value of 6.15 m. In the design with both the ATDB and the ACDB controllers, the maximum lateral displacement of the car body’s trajectory is 1 m, reducing to 83.6% from the baseline value of 6.1 m; and the maximum lateral displacement of the trailer body’s trajectory is 1 m, reducing to 83.7% from the baseline value of 6.15 m. All simulation results show that the ATDB and the ACDB controller are effective to improve the lateral stability of the CarSim model at the emergency evasive maneuver under low lateral acceleration.

Figure 4.23. Car body’s trajectory of the CarSim model with and without the controller under the emergency evasive maneuver at U=27.78 m/s
Figure 4.24. Trailer body’s trajectory of the CarSim model with and without the controller under the emergency evasive maneuver at $U=27.78$ m/s.

To quantitatively analyze the CT system lateral stability based on the cases of the designs with and without the ATDB and the ACDB controller, the dynamic responses of the CarSim model in the emergency evasive maneuver are summarized in Table 4.4.

Table 4.4 the dynamic responses of the CarSim model under the emergency evasive maneuver

<table>
<thead>
<tr>
<th>CarSim Low g (peak value)</th>
<th>Car lateral acceleration(g)</th>
<th>Trailer lateral acceleration(g)</th>
<th>Articulation angle (deg)</th>
<th>Car yaw rate(deg/s)</th>
<th>Trailer yaw rate(deg/s)</th>
<th>Car lateral displacement(m)</th>
<th>Trailer lateral displacement(m)</th>
<th>Car roll angle(deg)</th>
<th>Trailer roll angle(deg)</th>
</tr>
</thead>
<tbody>
<tr>
<td>CarSim-nc</td>
<td>0.4</td>
<td>0.48</td>
<td>2</td>
<td>8</td>
<td>15</td>
<td>6.1</td>
<td>6.15</td>
<td>1.1</td>
<td>0.8</td>
</tr>
<tr>
<td>CarSim-2c</td>
<td>0.2</td>
<td>0.28</td>
<td>2</td>
<td>7.5</td>
<td>4.5</td>
<td>5.2</td>
<td>5.2</td>
<td>0.7</td>
<td>0.49</td>
</tr>
<tr>
<td>Improvement</td>
<td>50%</td>
<td>41.7%</td>
<td>71.4%</td>
<td>50%</td>
<td>37.5%</td>
<td>17.3%</td>
<td>14.7%</td>
<td>36.4%</td>
<td>38.8%</td>
</tr>
<tr>
<td>CarSim-6c</td>
<td>0.08</td>
<td>0.1</td>
<td>1</td>
<td>2</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>0.25</td>
<td>0.08</td>
</tr>
<tr>
<td>Improvement</td>
<td>80%</td>
<td>79.2%</td>
<td>85.7%</td>
<td>86.7%</td>
<td>87.5%</td>
<td>83.6%</td>
<td>83.7%</td>
<td>77.3%</td>
<td>90%</td>
</tr>
</tbody>
</table>
As discussed above, in the emergency evasive maneuver at low lateral acceleration of CT system, based on the 3DOF-L, 4DOF-NL, 6DOF-NL and CarSim models, compared with the baseline design, the car-trailer system with the ATDB controller has better performance in terms of vehicle dynamic responses. The CarSim model with both the ATDB and the ACDB controllers has better performance than the CarSim model with only the ATDB controller. It indicates that at low lateral acceleration the new ATDB strategy based on the 3-DOF linear state model and the LQR technique can be used to control the nonlinear model, such as the 4DOF-NL and 6DOF-NL.

### 4.7.2 A Single Lane-Change Test Maneuver of High Lateral Acceleration

In this subsection, a single lane-change test maneuver at forward speed 27.78 m/s (100km/h) is used to examine the dynamic behaviors of all four CT models with the corresponding ATBD controller at high lateral acceleration of CT system. The car front wheel steering angle input is shown in Figure 4.25.
Figures 4.26-4.35 illustrate simulation results for the single lane-change test maneuver at high lateral acceleration of the CT system in terms of the time history of the car lateral acceleration, trailer lateral acceleration, articulation angle, car yaw rate, trailer yaw rate, car roll angle, trailer roll angle, car body’s trajectory and trailer body’s trajectory, respectively, using the 3DOF-L, 4DOF-NL, 6DOF-NL without and with the ATBD controller. As the simulation results shown: the ATDB controller does not function in the 4DOF-NL model case and the 6DOF-NL model case.

However, the ATDB controller still works in the 3DOF-L model case as shown in Figures 4.26-4.35. Compared with the baseline vehicle, the design with the controller based on the 3DOF-L model has superior performance: the maximum peak value of the car acceleration is 1 g, reducing 71.4% from the baseline value of 3.5 g; the maximum
The peak value of the trailer acceleration variation is 1.1 g, reducing 73.8% from the baseline value of 4.2 g; the maximum peak value of the articulation angle variation is 4 deg, reducing 80% from the baseline value of 20 deg; the maximum peak value of the car yaw rate variation is 25 deg/s, reducing 66.6% from the baseline value of 75 deg/s; the maximum peak value of the trailer yaw rate variation is 25 deg/s, reducing to 72.3% from the baseline value of 110 deg/s; the maximum lateral displacement of the car body’s trajectory is 30 m, reducing to 62% from the baseline value of 79 m; and the maximum lateral displacement of the trailer body’s trajectory is 30 m, reducing to 62.5% from the baseline value of 80 m. Note that in reality the 3DOF-L model without the ATDB can’t achieve the peak lateral acceleration as high as 3.5 g due to the limitation of the tire/road lateral force.

Figure 4.26. Car lateral acceleration versus time with and without the ATDB controller under the single lane change maneuver at U=27.78 m/s using the 3DOF-L, 4DOF-NL and 6DOF-NL models
Figure 4.27. Trailer lateral acceleration versus time with and without the ATDB controller under the single lane change maneuver at U=27.78 m/s using the 3DOF-L, 4DOF-NL and 6DOF-NL models.

Figure 4.28. Articulation angle versus time with and without the ATDB controller under the single lane change maneuver at U=27.78 m/s using the 3DOF-L, 4DOF-NL and 6DOF-NL models.
Figure 4.29. Articulation angle versus time with the ATDB controller under the single lane change maneuver at $U=27.78$ m/s using the 3DOF-L, 4DOF-NL and 6DOF-NL models.

Figure 4.30. Car yaw rate versus time with and without the ATDB controller under the single lane change maneuver at $U=27.78$ m/s using the 3DOF-L, 4DOF-NL and 6DOF-NL models.
Figure 4.31. Trailer yaw rate versus time with and without the ATDB controller under the single lane change maneuver at $U=27.78 \text{ m/s}$ using the 3DOF-L, 4DOF-NL and 6DOF-NL models.

Figure 4.32. Car roll angle versus time with and without the ATDB controller under the single lane change maneuver at $U=27.78 \text{ m/s}$ using the 6DOF-NL models.
Figure 4.33. Trailer roll angle versus time with and without the ATDB controller under the single lane change maneuver at U=27.78 m/s using the 6DOF-NL models.

Figure 4.34. Car body’s trajectory with and without the ATDB controller under the single lane change maneuver at U=27.78 m/s using the 3DOF-L, 4DOF-NL and 6DOF-NL models.
Figure 4.35. Trailer body’s trajectory with and without the ATDB controller under the single lane change maneuver at $U=27.78$ m/s using the 3DOF-L, 4DOF-NL and 6DOF-NL models.

To quantitatively analyze the CT system lateral stability based on the cases of the designs with and without the ATDB controller, the dynamic responses of the 3DOF-L model under the single lane change maneuver are summarized in table 4.5.

**Table 4.5** the dynamic responses of the 3DOF-L model under the single lane change maneuver

<table>
<thead>
<tr>
<th>3DOF-L High g (peak value)</th>
<th>Car lateral acceleration(g)</th>
<th>Trailer lateral acceleration(g)</th>
<th>Articulation angle (deg)</th>
<th>Car yaw rate(deg/s)</th>
<th>Trailer yaw rate(deg/s)</th>
<th>Car lateral displacement(m)</th>
<th>Trailer lateral displacement(m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>3DOF-L-n3</td>
<td>3.5</td>
<td>4.2</td>
<td>20</td>
<td>75</td>
<td>110</td>
<td>79</td>
<td>80</td>
</tr>
<tr>
<td>3DOF-L-I3</td>
<td>1</td>
<td>1.1</td>
<td>4</td>
<td>25</td>
<td>25</td>
<td>30</td>
<td>30</td>
</tr>
<tr>
<td>Improvement</td>
<td>71.4%</td>
<td>73.8%</td>
<td>80%</td>
<td>66.6%</td>
<td>72.3%</td>
<td>62%</td>
<td>62.5%</td>
</tr>
</tbody>
</table>
For the baseline vehicle and the design based on the CarSim model with the ATDB controller and with both the ATBD and the ACBD controllers, Figures 4.36-4.44 illustrate simulation results for the single lane-change test maneuver at high lateral acceleration of the CT system in terms of the time history of the car lateral acceleration, trailer lateral acceleration, articulation angle, car yaw rate, trailer yaw rate, car roll angle, trailer roll angle, car body’s trajectory and trailer body’s trajectory, respectively. The simulation results indicate that the ATDB controller doesn’t work in the CarSim model case, but if we add the ACDB controller in the control strategy, the unstable motion model will be effectively prevented in the CarSim model.

With the new control strategy based on both the ATBD and the ACBD controllers, the simulation results are shown in the figure 4.36-4.44: the maximum peak value of the car acceleration variation is 0.3 g, reducing 62.5% from the baseline value of 0.8 g; the maximum peak value of the trailer acceleration variation is 0.22 g, reducing 68.6% from the baseline value of 0.7 g; the maximum peak value of the articulation angle variation is 2 deg, reducing 85.7% from the baseline value of 14 deg; the maximum peak value of the car yaw rate variation is 5 deg/s, reducing 84.6% from the baseline value of 32 deg/s; the maximum peak value of the trailer yaw rate variation is 7 deg/s, reducing 80.5% from the baseline value of 36 deg/s; the maximum peak value of the car roll angle variation is 0.6 deg, reducing 73.3% from the baseline value of 2.25 deg; the maximum peak value of the trailer roll angle variation is 0.4 deg, reducing by 69.2% from the baseline value of 1.3
the maximum lateral displacement of both the car body’s trajectory the trailer body’s trajectory is 6 m.

Figure 4.36. Car lateral acceleration versus time of the CarSim model with and without the controller under the single lane change maneuver at U=27.78 m/s

Figure 4.37. Trailer lateral acceleration versus time of the CarSim model with and without the controller under the single lane change maneuver at U=27.78 m/s
Figure 4.38. Articulation angle versus time of the CarSim model with and without the controller under the single lane change maneuver at $U=27.78$ m/s

Figure 4.39. Car yaw rate versus time of the CarSim model with and without the controller under the single lane change maneuver at $U=27.78$ m/s
Figure 4.40. Trailer yaw rate versus time of the CarSim model with and without ATDB controller of the single lane change maneuver at U=27.78 m/s

Figure 4.41. Car roll angle versus time of the CarSim model with and without the controller under the single lane change maneuver at U=27.78 m/s
Figure 4.42. Trailer roll angle versus time of the CarSim model with and without the controller under the single lane change maneuver at U=27.78 m/s

Figure 4.43. Car body’s trajectory of the CarSim model with and without the controller under the single lane change maneuver at U=27.78 m/s
Figure 4.44. Trailer body’s trajectory of the CarSim model with and without the controller under the single lane change maneuver at \( U = 27.78 \text{ m/s} \)

To quantitatively analyze the CT system lateral stability based on the cases of the designs with and without the ATDB and the ACDB controller, the dynamic responses of the CarSim model under the single lane change maneuver are summarized in Table 4.6.

**Table 4.6 the dynamic responses of the CarSim model under the single lane change maneuver**

<table>
<thead>
<tr>
<th>CarSim Hich g (peak value)</th>
<th>Car lateral acceleration(g)</th>
<th>Trailer lateral acceleration(g)</th>
<th>Articulation angle (deg)</th>
<th>Car yaw rate(deg/s)</th>
<th>Trailer yaw rate(deg/s)</th>
<th>Car lateral displacement(m)</th>
<th>Trailer lateral displacement(m)</th>
<th>Car roll angle(deg)</th>
<th>Trailer roll angle(deg)</th>
</tr>
</thead>
<tbody>
<tr>
<td>CarSim-nc</td>
<td>0.8</td>
<td>0.7</td>
<td>14</td>
<td>32</td>
<td>36</td>
<td>understeer</td>
<td>understeer</td>
<td>2.25</td>
<td>1.3</td>
</tr>
<tr>
<td>CarSim-6c</td>
<td>0.3</td>
<td>0.22</td>
<td>2</td>
<td>5</td>
<td>7</td>
<td>6</td>
<td>6</td>
<td>0.6</td>
<td>0.4</td>
</tr>
<tr>
<td>Improvement</td>
<td>62.5%</td>
<td>68.6%</td>
<td>85.7%</td>
<td>84.6%</td>
<td>80.5%</td>
<td>100%</td>
<td>100%</td>
<td>73.3%</td>
<td>69.2%</td>
</tr>
</tbody>
</table>
Figure 4.45. Lateral force of the trailer wheel versus time of the 3DOF-L model with and without the controller under the single lane change maneuver at U=27.78 m/s

Figure 4.46. Trailer tire side-slip angle vs. lateral force of the trailer wheel of the 3DOF-L model with and without the controller under the single lane change maneuver at U=27.78 m/s
Figure 4.47. Lateral force of the trailer wheel versus time of the 4DOF-NL model with and without the controller under the single lane change maneuver at \( U=27.78 \text{ m/s} \).

Figure 4.48. Trailer tire side-slip angle vs. lateral force of the trailer wheel of the 4DOF-NL model with and without the controller under the single lane change maneuver at \( U=27.78 \text{ m/s} \).
Figure 4.45-4.48 show the relationship between the trailer tire lateral tire force and the trailer tire side-slip angle based on the 3DOF linear and the 4DOF nonlinear models with and without the ATDB controller. As the simulation results shown, in the case of the 3DOF-L model, the lateral force of the trailer tire increases with the side-slip angle, and the ATDB controller can effectively enhance the lateral stability of CT systems. However, in the case of the 4DOF-NL model, the simulation results based on the nonlinear tire model shows that: the tire side-slip angle increase with time, but the trailer lateral tire force will be saturated once the tire side-slip angle over approximately 7 deg and the lateral force will be saturated at approximately 5.9 KN. In this case, the ATDB controller is not working in the 4DOF-NL model.

4.8. SUMMARY

This chapter presents a new active trailer differential braking (ATDB) controller based on the LQR technique. The ATDB controller is examined and evaluated in terms of the lateral stability of car-trailer (CT) systems. The ATDB controller is designed using the 3-DOF linear model, then the new ATDB controller is applied to the nonlinear 4-DOF, the nonlinear 6-DOF and the 21-DOF CT (CarSim) models using a set control gains which are based on the 3-DOF linear model and the LQR technique. In the CarSim case, the ATDB controller is combined with the active car differential braking (ACDB) to control the yaw motions of both the car and the trailer. According to the analysis of the simulation results the insightful findings are summarized as follows:
(1) Under the regular evasive maneuver at low lateral accelerations of car and trailer bodies (less than 0.5 g), the lateral stability of the four models can be improved with the application of the ATDB controller. In the CarSim model case, the ATDB controller is not working as well as in the other three models. However, if we introduce the ACDB controller, the CarSim model will have better performance than the other three models. Numerical simulations indicate that the ATBD control strategy based on a linear model and the LQR technique can be used to control the lateral stability of nonlinear models when the lateral accelerations of car and trailer bodies are less than 0.5 g. To further improve the lateral stability of the CT system, the ATDB and the ACDB controllers should be integrated. Thus, the yaw motions of both the car and the trailer can be controlled.

(2) Under the single lane change maneuver at high lateral low lateral accelerations of car and trailer bodies (larger than 0.5g), the linear model-based the ATDB controller can’t effectively enhance the lateral stability of CT systems. Under high later acceleration of the car, the ATDB controller installed on the trailer can’t effectively control the lateral motions of the car. With the combination of the ATDB and the ACDB controllers, the lateral stability of the CT system can be enhanced, even through the lateral acceleration of the car and the trailer take large values, e.g., larger than 0.5 g.
Chapter 5

CONCLUSIONS

5.1 INTRODUCTION

As discussed and identified in Chapter 1, the two ultimate objectives of this thesis are to examine the dynamic behaviors of the typical linear and nonlinear vehicle models and to develop a novel method for the design of car-trailer (CT) systems with active trailer differential braking (ATDB) systems. The numerical simulation results indicate that the goals have been successfully achieved. The feasibility and efficacy of the method has been demonstrated though the investigation of the lateral stability of a CT system. This design synthesis method and the numerous conclusions drawn from the above numerical experiments are believed to be important contributions to the design optimization of CT systems with ATDB systems.

The achievements of the research and the scope of future research are addressed in this chapter.

5.2 COMPARISON NUMERICAL MODELS

In this thesis, a 3 degrees of freedom (DOF) linear model, a 4-DOF nonlinear model and a 6-DOF nonlinear model are generated and compared against a multibldy system model with 21-DOF developed in CarSim commercial software in terms of fidelity, complexity,
and applicability for lateral motion controller design. All four models are compared in terms of their dynamic performance measures under while performing a single lane change maneuver at a low lateral acceleration and a high lateral acceleration. Based on the comparison and discussion of the simulation results, the conclusions can be drawn as follows:

1. The linear model with 3-DOF can be used to predict the lateral stability (critical speed and unstable motion modes) in the initial design of CT systems.

2. When the lateral acceleration of CT system is small, such as 0.5 g, the linear model will provide dynamic responses that are in good agreement with the nonlinear vehicle models, and this linear yaw/plane model can be efficiently used for the lateral motion controller design under low lateral acceleration maneuvers.

3. When the lateral acceleration of CT system is larger than 0.5 g, the linear model will not provide dynamic responses that are not in good agreement with the nonlinear vehicle models, and this linear yaw/plane model is not suitable for the lateral stability controller design.

4. Whether the lateral acceleration of CT system is small or larger, the dynamic responses for the 4DOF-NL, 6DOF-NL and CarSim models agree each other very well. Thus, it is demonstrated that compared with multibody system model with 21-DOF, the nonlinear yaw/roll model with 6-DOF is effective in terms of fidelity, complexity, and computational efficiency. The 3-dimensional model with 6 DOF
can be used for lateral stability controller design under maneuvers considering the roll motions of the car and the trailer.

5.3 ACTIVE TRAILER DIFFERENTIAL BRAKING CONTROLLER

The research also developed and tested an active trailer differential braking (ATDB) controller to improve the lateral stability of CT systems. The ATDB controller is derived using the Linear Quadratic Regular (LQR) technique and is based on the 3-DOF linear model. Then, the new ATDB strategy is tested using for the nonlinear 4-DOF, nonlinear 6-DOF, and 21-DOF CT (CarSim) models. In the CarSim case, in addition to the ATDB controller, another ACDB (Active Car Differential Braking) controller is introduced to control the car yaw moment. Based on the analysis of the simulation results, the proposed controller bears the following features:

(1) Under the regular evasive maneuver at low lateral accelerations of car and trailer bodies (less than 0.5 g), the lateral stability of the four models can be improved with the application of the ATDB controller. In the CarSim model case, the ATDB controller is not working as well as in the other three models. However, if we introduce the ACDB controller, the CarSim model will have better performance than the other three models. Numerical simulations indicate that the ATBD control strategy based on a linear model and the LQR technique can be used to control the lateral stability of nonlinear models when the lateral accelerations of car and trailer bodies are less than 0.5 g. To further improve the
lateral stability of the CT system, the ATDB and the ACDB controllers should be integrated. Thus, the yaw motions of both the car and the trailer can be controlled.

(2) Under the single lane change maneuver at high lateral low lateral accelerations of car and trailer bodies (larger than 0.5 g), the linear model-based the ATDB controller can’t effectively enhance the lateral stability of CT systems. Under high later acceleration of the car, the ATDB controller installed on the trailer can’t effectively control the lateral motions of the car. With the combination of the ATDB and the ACDB controllers, the lateral stability of the CT system can be enhanced, even through the lateral acceleration of the car and the trailer take large values, e.g., larger than 0.5 g.

5.4 DIRECTIONS FOR FUTURE RESEARCH

To improve the proposed design synthesis method for CT systems with ATDB systems, the following directions for future research are recommended:

(1) Nonlinear hitch models should be developed and included in vehicle models to test the rollover performance of CT systems.

(2) A driver model should be developed and included in the vehicle model to simulate the closed-loop testing maneuvers.

(3) In this thesis, the set of control gains is based on the 3-DOF linear model and the LQR technique at the given vehicle forward speeds. To increase robustness of the
ATDB controller, the control gain scheduling scheme should be used, which considers the varied operating conditions of the vehicle, such as vehicle speed and payload. Thus, a design optimization method should be developed to optimize the LQR controller for any practical vehicle speed and maneuver.
REFERENCES


[34] Shuwen, Z., Siqi, Z., Guangyao, Z. and Chuanyin, T. 2010. Lateral Stability Control of Car-trailer Combination Based on 4WS, *International Conference on*


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APPENDIX

APPENDIX A: MATRICES FOR THE LINEAR MODEL WITH 3DOF

\[
M = \begin{bmatrix}
    m_1 + m_2 & -m_2 (d_1 + c) & m_2 c & 0 \\
    -m_2 d_1 & I_1 + m_2 d_1 (d_1 + c) & -m_2 c d_1 & 0 \\
    -m_2 c & I_2 + m_2 c (d_1 + c) & -I_2 - m_2 c^2 & 0 \\
    0 & 0 & 0 & 1 \\
\end{bmatrix}
\]

\[
D = \frac{1}{\mu} \begin{bmatrix}
    c_r + c_d + c_i & c_s a - c_s b - c_i (d_1 + c + d_2) - (m_1 + m_2) \mu^2 & c_s (d_1 + c) & c_s \mu \\
    c_s a - c_s b - c_d & c_s a^2 + c_s b^2 + c_i d_1 + c + d_1 + m_1 d_1 \mu^2 & -c_s d_1 d_2 & -c_s d_1 \mu \\
    -c_s (d_1 + c) & c_s (d_1 + c + d_2) (d_1 + c) + m_2 c \mu^2 & -c_s (d_2 + c)^2 & -c_s (d_2 + c) \mu \\
    0 & 0 & \mu & 0 \\
\end{bmatrix}
\]

\[
F = \begin{bmatrix}
    -c_f \\
    -c_f a \\
    0 \\
    0 \\
\end{bmatrix}; \quad C_B = \begin{bmatrix}
    0 \\
    0 \\
    -1 \\
    0 \\
\end{bmatrix}
\]

APPENDIX B: GOVERNING EQUATIONS FOR THE NONLINEAR MODEL WITH 4DOF

\[
\begin{bmatrix}
    m_1 + m_2 & 0 & m_1 d_2 \sin(\psi_2) & 0 \\
    0 & m_1 + m_2 & -m_1 (d_1 + d_2 \cos(\psi_2)) & -m_1 d_2 \cos(\psi_2) \\
    m_1 d_2 \sin(\psi_2) & -m_1 (d_1 + d_2 \cos(\psi_2)) & \left(I_c + I + 2m_1 d_2 \cos(\psi_2) + m_1 d_1^2 \right); & 0 \\
    m_1 d_2 \sin(\psi_2) & -m_1 d_2 \cos(\psi_2) & I_c + m_1 d_2 \cos(\psi_2) + m_1 d_2^2 & I_c + m_1 + d_2^2 \\
\end{bmatrix}\begin{bmatrix}
    \dot{U} \\
    \dot{V} \\
    \dot{\psi}_1 \\
    \dot{\psi}_2 \\
\end{bmatrix}
\]
\[
\begin{aligned}
&\left( m_1 + m_2 \right) V \psi_1 - m_1 d_1 \psi_1^2 - m_2 d_2 \cos(\psi_2)(\psi_1 + \psi_2)^2 + X_f \cos(\delta_f) + X_1 \cos(\delta_1) + X_2 \cos(\psi_2) \\
&- Y_f \sin(\delta_f) - Y_1 \sin(\delta_1) - Y_2 \sin(\psi_2);
\end{aligned}
\]
\[
\begin{aligned}
&-(m_1 + m_2) U \psi_1 - m_1 d_1 \sin(\psi_2)(\psi_1 + \psi_2)^2 + X_f \sin(\delta_f) + X_1 \sin(\delta_1) + X_2 \sin(\psi_2) \\
&+ Y_f \cos(\delta_f) + Y_1 \cos(\delta_1) + Y_2 \cos(\psi_2);
\end{aligned}
\]
\[
\begin{aligned}
&m_2 d_2 (V \sin(\psi_2) + U \cos(\psi_2)) \psi_1 + m_1 d_1 \psi_1 + m_2 d_2 \sin(\psi_2)(\psi_1 + \psi_2)^2 + 2\psi_1 \psi_2 \\
&+ aX_f \sin(\delta_f) - bX_1 \sin(\delta_1) - d_1 X_2 \sin(\psi_2) + aY_f \cos(\delta_f) - bY_1 \cos(\delta_1) - (d_2 + c + d_1 \cos(\psi_2)) Y_2; \\
&m_1 d_2 (V \sin(\psi_2) + U \cos(\psi_2)) \psi_1 - m_1 d_1 \psi_1 + m_2 d_2 \sin(\psi_2) \psi_1^2 - (d_2 + c) Y_2;
\end{aligned}
\]