DESIGN AND DEVELOPMENT OF A NOVEL LIGHTWEIGHT LONG-REACH COMPOSITE ROBOTIC ARM

by

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Bachelor of Engineering, University of Ontario Institute of Technology, 2007

A THESIS SUBMITTED IN PARTIAL FULFILLMENT OF THE REQUIREMENTS FOR THE DEGREE OF

Masters of Applied Science

in

The Faculty of Engineering and Applied Science

Mechanical Engineering

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THE UNIVERSITY OF ONTARIO INSTITUTE OF TECHNOLOGY

June 2009

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ABSTRACT

Metallic robotic arms, or manipulators, currently dominate automated industrial operations, but due to their intrinsic weight, have limited usefulness for large-scale applications in terms of precision, speed, and repeatability. This thesis focuses on exploring the feasibility of using polymeric composite materials for the construction of long-reach robotic arms. Different manipulator layouts were investigated and an ideal design was selected for a robotic arm that has a 5 [m] reach, 50 [kg] payload, and is intended to operate on large objects with complex curvature.

The cross-sectional geometry of the links of the arm were analyzed for optimal stiffness- and strength-to-weight ratios that are capable of preserving high precision and repeatability under time-dependent external excitations. The results lead to a novel multi-segment link design and method of production.

A proof-of-concept prototype of a two degrees-of-freedom (2-DOF) robotic arm with a reach of 1.75 [m] was developed. Both static and repeatability testing were performed for verification. The results indicated that the prototype robot main-arm constructed of carbon fiber-epoxy composite material provides good stiffness-to-weight and strength-to-weight ratios. Finite element analysis (FEA) was performed on a 3-D computer model of the arm. Successful verification led to the use of the 3-D model to define the dimensions of an industrial-sized robotic arm. The results obtained indicate high stiffness and minimal deflection while achieving a significant weight reduction when compared to commercial arms of the same size and capability.
Dedication

To my father Garth, my mother Lynne, and all my friends and family who always supported my efforts.
Acknowledgements

First and foremost I would like to thank my advisors, Dr. Scott Nokleby and Dr. Remon Pop-Iliev, who have provided me with continued support throughout my graduate career. In addition to this, I am grateful for their unending guidance throughout all aspects of my thesis work. Their advice will prove to be valuable in my future endeavours and for this their efforts will not be forgotten.

I would like to thank Mr. James Beltrano, Mr. Steven Bemis, Mr. Mike MacLeod, and Mr. Garth Willis. Without their help I would not have successfully constructed my prototype.

I also feel the need to acknowledge John Damiani, Jeff Martin, Kyle Mullin, Brad Timson, and David Tripp. Their assistance throughout varying aspects of this project as part of their undergraduate theses is greatly appreciated. Additionally, I would also like to thank fellow graduate students Emad Abdalla, Michael Frejek, Sasha Ginzburg, and Brian Riess who were always willing to offer their assistance.

I would like to thank BRIC Engineered Systems, the Ontario Centres of Excellence (OCE), and the Natural Sciences and Engineering Research Council (NSERC) of Canada whose financial support has made my research possible.
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Nomenclature

CAN  computer area network
CFRP  carbon fiber reinforced plastic
DOF  degrees-of-freedom
FEA  finite element analysis
MDF  medium density fiberboard
MEKP  methyl ethyl ketone peroxide
PAN  polyacrylonitrile
PD  proportional and derivative
R  revolute joint
SCARA  selectively compliant assembly robot arm
US  United States
Chapter 1

Introduction

1.1 Problem Definition and Goal of Research

Currently, robotic arms that are used for large-scale applications are constructed of either steel or aluminum. These materials, however, have a low specific stiffness (stiffness/density) and specific strength (strength/density) and thus the respective robotic arms feature a poor payload-to-weight ratio. Therefore, the robotic arms are typically very large and heavy in order to provide high precision and repeatability while being rated for the required payload. Consequently, very powerful and expensive motors are necessary to provide movement for such heavy robotic arms.

Due to the intrinsic weight of steel and aluminum arms, there is a trade-off between providing high stiffness and lightweight. Therefore, a material with higher specific stiffness and strength needs to be implemented into the construction of robotic arms.

The goal of this research is to investigate the use of composite materials in robot arm construction. In addition to this, in order to offer further improvements over current industrial-sized robotic arms in terms of both the stiffness and load carrying capacity of the arm while decreasing the weight, the design of lightweight robotic links is investigated thoroughly in this research work.
1.2 Customer Requirements

The objective of this thesis was to design a 6-DOF robotic arm capable of large-scale applications which met or exceeded the following functional requirements:

- As light as possible.
- Payload of 50 [kg].
- Reach of 5 [m].
- Repeatability equivalent to industry standard.
- Able to fit through a 0.76 [m] (30 [in]) diameter opening.
- Tool linear speed of 0.15 [m/s] (6 [in/s]).
- Capable of performing operations on large structures with complex curvature.

One such application of the robotic arm is performing operations on airplanes. Consequently, the robotic arm must be designed for high maneuverability in order to navigate the geometry of the plane. Additionally, since airplanes are very large structures, the entire robotic arm must be able to move around the periphery of the airplane. As a result, the emphasis on creating a lightweight robotic arm is of even greater concern.

1.3 Typical Properties of Traditional Commercial Robotic Arms

Currently, there are no commercially available robotic arms that can satisfy the robot specifications. As far as 6-DOF commercial robotic arms that are rated for a payload of approximately 50 [kg] are concerned, they have a maximum reach of approximately 2 [m]. Therefore, to have a better idea of the specifications of competitor robotic
arms, longer reach arms (that subsequently have a higher rated payload) were also investigated. Table 1.1 outlines some of the primary specifications for five commercial robotic arms.

<table>
<thead>
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<th>PROPERTY</th>
<th>Kuka KR 60-3</th>
<th>Fanuc M-710iC/50</th>
<th>ABB IRB 4600-60</th>
<th>Kuka KR 100-2P</th>
<th>Motoman EPH100</th>
</tr>
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<tbody>
<tr>
<td>Mass</td>
<td>635 kg</td>
<td>560 kg</td>
<td>435 kg</td>
<td>1465 kg</td>
<td>1520 kg</td>
</tr>
<tr>
<td>Payload</td>
<td>60 kg</td>
<td>50 kg</td>
<td>60 kg</td>
<td>100 kg</td>
<td>100 kg</td>
</tr>
<tr>
<td>Reach</td>
<td>2033 mm</td>
<td>2050 mm</td>
<td>2050 mm</td>
<td>3500 mm</td>
<td>3010 mm</td>
</tr>
<tr>
<td>Repeatability</td>
<td>±0.20 mm</td>
<td>±0.07 mm</td>
<td>±0.19 mm</td>
<td>±0.20 mm</td>
<td>±0.30 mm</td>
</tr>
<tr>
<td>Axis 1 Speed</td>
<td>128°/s</td>
<td>175°/s</td>
<td>175°/s</td>
<td>102°/s</td>
<td>110°/s</td>
</tr>
<tr>
<td>Axis 2 Speed</td>
<td>102°/s</td>
<td>175°/s</td>
<td>175°/s</td>
<td>96°/s</td>
<td>110°/s</td>
</tr>
<tr>
<td>Axis 3 Speed</td>
<td>128°/s</td>
<td>175°/s</td>
<td>175°/s</td>
<td>95°/s</td>
<td>110°/s</td>
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The first three robotic arms that have a payload and reach of approximately 50 [kg] and 2 [m], respectively, have a mass ranging from 435 [kg] to 635 [kg]. Consequently, their payload-to-weight ratio is approximately 0.10. These robotic arms only have a reach of 2 [m], so in order to provide a reach of 5 [m] the payload-to-weight ratio would be considerably worse. The larger arms that have a payload of 100 [kg] and a reach of 3.0 [m] and 3.5 [m], have an even lower payload-to-weight ratio of approximately 0.066. Therefore, it can be assumed that a similar robot that has a payload of 50 [kg] and a reach of 5 [m] would feature a payload-to-weight ratio of approximately 0.05. Therefore, the total weight of the robotic arm would be in the range of 1,000 [kg]. Additionally, the repeatability quoted in Table 1.1 is generally around ±0.20 mm, therefore the robotic arm is designed to this specification.

All the robotic arms seem to have very high motor speeds. Motor speeds this high are required for pick-and-place operations but for the intended application these speeds are unnecessary. Thus, less powerful, and consequently lighter, motors can be used to operate at lower speeds in order to reduce the overall weight of the robotic arm.

The problem with current large-scale application robotic arms is that they are very
heavy. Consequently, the motors and drive systems that are required for the robot to operate at the desirable levels of speed and accuracy are very powerful and thus expensive. In order to alleviate this problem, a lighter alternative to the heavy steel and aluminum arms that are currently used must be realized. Polymer composites are potentially a feasible material choice for constructing a lightweight large-scale robotic arm that still offers long reach, high payload, and good repeatability.

1.4 Review of Polymer Composites

1.4.1 Overview

Polymer composites can be created by combining two distinct polymer materials to obtain a material that has a combination of properties. Consequently, polymer composites can be made to offer stiffness and strength that is better than aluminum and competitive with steel even though they are around five times lighter than steel and half the mass of aluminum. Consequently, polymer composites are about eight to ten times better than steel and aluminum in terms of specific stiffness and specific strength [5]. Therefore, this research work focuses on exploring the feasibility of using polymeric composite materials for constructing lightweight long-reach robotic arms.

1.4.2 Particle-Reinforced Composites

A particle-reinforced composite is a composite in which the dispersed material has a sphere-like shape with similar dimensions in all directions. The matrix material is continuous and surrounds these particles. The properties of composites are not only a function of the properties of the materials and their relative amounts, but also a function of the geometry of the dispersed material within the matrix material. The characteristics of the dispersed phase geometry are the particle size, shape, distribution, and orientation as shown in Figure 1.1 [5].
Particle-reinforced composites are classified as either large particle composites or dispersion-strengthened composites. Large particle composites are defined as composites in which the particle is so large that the particle-matrix interactions cannot be treated on the atomic or molecular level [5]. The particles restrain movement of the matrix material at the locations at which the particles exist. The particles bear some of the load because the matrix transfers some of the applied stress to the particles. Due to the size of the particles dispersed within the matrix material, the amount of reinforcement depends on the strength of the bonds at the matrix-particle interface. The mechanical properties of the composite are influenced by the volume fraction of the matrix and the particle. The mechanical properties of the composite improve as the volume fraction of the dispersed particles is increased. In fact, the mechanical properties of the composite can be predicted within an upper and lower bound as demonstrated in Figure 1.2 if the mechanical properties and volume fractions of the two materials are known [5].

The particles within dispersion-strengthened composites are considerably smaller such that the particle-matrix interactions that lead to strengthening occur on the atomic or molecular level [5]. Consequently, the matrix bears the majority of an applied load and the particles impede the motion of dislocations within the matrix, restricting plastic deformation. These composites prove to offer greater improvements in the mechanical behaviour than large particle composites but due to the small size of the dispersed particles the fabrication of these composites is more complicated [5].
1.4.3 Fiber-Reinforced Composites

1.4.3.1 Mechanical Properties

A fiber-reinforced composite is a composite in which the dispersed material is in the form of a fiber (i.e., the length is considerably greater than the diameter). Fiber-reinforced composites are categorized by the length of the fiber. Those designated as continuous fiber-reinforced composites are composites in which the fibers are long, whereas discontinuous fiber-reinforced composites are composites in which the fibers are short. In fact, a continuous fiber has a length that is equal to or greater than fifteen times that of the critical length and a discontinuous fiber has a length shorter than this. The critical fiber length, $l_c$, is described as being the fiber length necessary for effective strengthening and stiffening of the composite material and can be defined as [5]:

$$ l_c = \frac{\sigma_f^* d}{2\tau_c} $$  \hspace{1cm} (1.1)

where $\sigma_f^*$ is the fiber ultimate or tensile strength, $d$ is the fiber diameter, and $\tau_c$ is the smaller value of the fiber-matrix bond strength and the shear yield strength of the matrix.
The mechanical characteristics of the fiber-reinforced composite depend on the ability of the composite to transmit the applied load from the matrix material to the fibers. The interfacial bond between the fiber and matrix material acts as the medium. The fiber-matrix bond is eliminated at the ends of the fibers when a stress is applied and thus there is no load transmittance there. As a result, a fiber-reinforced composite with an equal volume fraction of the fiber material but longer fibers than a composite of the same two materials will be able to provide more load transmittance to the fibers since the ends of the fibers contribute to a lower percentage of the total volume of the fiber material. This effect is illustrated in Figure 1.3a. When the length of the fiber is equal to the critical length the maximum fiber load is achieved only at the middle of the fiber. However, for fiber lengths that are greater than the critical length, the maximum fiber load is achieved over a greater length of the fiber, thus increasing the amount of load that is transmitted to the fiber. This translates to an increase in the amount of reinforcement that is provided. However, the shorter the fiber the less inherent flaws there will be in the material and thus the closer the actual strength of the material will be to the theoretical value. Therefore, in order to benefit from using continuous fibers, it is best to use a fiber with a small diameter because as the fibers become smaller in diameter, the chances of inherent flaws in the material are reduced [6].

The stress-strain diagram for the fiber, matrix and resulting composite material is shown in Figure 1.3b. As it illustrates, once the applied stress has surpassed the yield stress of the matrix material the composite material does not undergo plastic deformation but rather the rate of elastic deformation is just increased. As the matrix material begins to undergo plastic deformation the fibers that reinforce the composite support more of the applied load and stretch elastically at a greater rate until the strain has reached the tensile strain of the fiber. At this point, failure of the composite material occurs. However, the failure is not catastrophic. After the fibers fail, the
matrix is still intact since its tensile strain is greater than that of the fibers. Consequently, the composite now contains shortened fibers that are capable of sustaining a diminished load as the matrix continues to plastically deform [5].

![Fiber Stress-Position Profiles, Polymer Composite Stress-Strain Curve](image)

Figure 1.3: Fiber Stress-Position Profiles, Polymer Composite Stress-Strain Curve [5]

Polymer composites are highly anisotropic materials in that their properties are dependent on the direction in which they are measured. When the fibers are aligned in one direction this anisotropic property is most evident. Since continuous fibers are always aligned, they produce polymer composites that are highly anisotropic. For this reason it is important to determine the mechanical properties under conditions of both longitudinal and transverse loading [5].

Under longitudinal loading the load is applied in the same direction as the fiber alignment direction. Under this condition it is assumed that the fiber-matrix interfacial bond is strong enough to ensure that the deformation of the fibers and matrix is the same. The modulus of elasticity of the polymer composite under longitudinal loading, $E_{cl}$, can be determined using:
\[ E_{ct} = E_m V_m + E_f V_f \]  \hspace{1cm} (1.2)

where \( E_m \) is the modulus of elasticity of the matrix material, \( E_f \) is the modulus of elasticity of the fiber, \( V_m \) is the volume fraction of the matrix material, and \( V_f \) is the volume fraction of the fiber [5].

For transverse loading, the load is applied in a direction perpendicular to the direction of fiber alignment. In this situation, the stress that the fiber and matrix material is exposed to is the same. As a result, the modulus of elasticity of the polymer composite under transverse loading, \( E_{ct} \), can be determined using [5]:

\[ E_{ct} = \frac{E_m E_f}{E_f V_m + E_m V_f} \]  \hspace{1cm} (1.3)

Under longitudinal loading the mechanical properties are the greatest whereas they are at their lowest while under transverse loading. Therefore, when the loading is applied in any other direction, the mechanical properties of the polymer composite falls somewhere between these two extremes.

Since the mechanical properties are maximized when the stress is applied in the same direction as the alignment of the fibers, the degree of reinforcement for different scenarios is compared to that of longitudinal loading. Despite the fact that the mechanical properties of the polymer composite are improved over that of the matrix material while under transverse loading, the improvement is so negligible that the reinforcement efficiency is assumed to be zero. For situations where the stress direction is changing it is better to have the fibers randomly oriented throughout the matrix material. This helps to ensure that the mechanical properties are isotropic (constant in all directions). If the fibers are randomly and uniformly distributed within a specific plane, then the reinforcement efficiency is \( 3/8 \) for a stress direction in any direction in the plane of the fibers. If the fibers are randomly and uniformly distributed within
all three dimensions, then the reinforcement efficiency is $1/5$ for a stress applied in any direction [5].

1.4.3.2 Fiber Materials

There are three primary fiber materials that are used as reinforcement in polymer composites. These are glass, aramid, and carbon. Glass is the most commonly used fiber in polymer composites because it is relatively inexpensive and easy to produce. It is easily drawn into fibers from a melt of glass in the molten state. The molten glass passes through numerous platinum alloy nozzles to produce filaments of glass. These filaments are then sprayed with an organic sizing solution that contains binders to help form the filaments into strands, lubricants to prevent abrasion of filaments, and coupling agents to improve adhesion between the glass fiber and the matrix material. The filaments are then wound together to form a strand. Glass fibers are also favoured because of their high strength. As a result, when embedded in a plastic matrix, a composite with a very high specific strength is created. However, despite their strength, glass fibers have a low elastic modulus. Consequently, they produce a composite that is not very stiff. They do however produce composites that are chemically inert making them useful in a variety of corrosive environments but they do degrade at temperatures above $200 \, ^\circ\text{C}$. Lastly, their density is considerably higher than the other two fiber materials, making them less desirable for the specified application.

Aramid fibers are an aromatic organic compound made of carbon, hydrogen, oxygen, and nitrogen. They are distinguished by their high strength and high modulus of elasticity despite having a low density. Their strength is especially high (the highest of the three fibers) making them good candidates for composites to be used for structural purposes. In addition to this, they have high impact, creep, and fatigue resistance. Furthermore they are a bit more than 10 times as expensive as glass fibers, as indicated
in Table 1.2, but are still relatively inexpensive. The manufacturing processes for the fibers is relatively easy as a solution of the necessary polymers and strong acids are extruded into hot cylinders then washed and dried on spools. Afterwards, the fibers are stretched and drawn to improve strength and stiffness. The major drawback of aramid fibers is that they are relatively weak in compression. For this reason, they should be avoided for applications that apply a compressive load.

Carbon fibers have an extremely high modulus of elasticity (much higher than that of aramid and glass fibers) and high strength. They are not really affected by temperature as they have a low thermal expansion and retain their high stiffness and strength at elevated temperatures. Their density is still relatively low, falling between that of aramid and glass fibers as shown in Table 1.2. Like the other two fiber materials mentioned, they are not affected by a wide variety of solvents, acids, and bases. However, their main drawback is their cost. Carbon fibers are considerably more expensive than glass fibers costing upwards of 80 times as much [5].

Carbon fibers are also advantageous since there is a vast diversity of different forms of carbon which enables the user to select the required strength and stiffness by using different organic precursor materials. In addition to this, the carbon fiber can be totally crystalline in a form commonly known as graphite. This further enhances the mechanical properties of the fiber. Using a polyacrylonitrile (PAN) precursor the carbon fibers are made as follows. PAN fibers are stretched five to ten times their length to improve their mechanical properties before being heated to 250 °C in order to stabilize their dimensions during the next heating processes. It is then heated to between 1,000 °C and 1,500 °C so that the complex organic substance can decompose to a simpler structure. The fibers are then surface treated to create carbon fibers. If graphite fibers are to be created, then before surface treating the fibers are heated once more to above 2,500 °C enabling the microstructures to become graphitic.
### Table 1.2: Properties of Three Fiber-Epoxy Composites [5]

<table>
<thead>
<tr>
<th></th>
<th>Glass $V_f=0.6$ (E-Glass)</th>
<th>Aramid $V_f=0.6$ (Kevlar 49)</th>
<th>Carbon $V_f=0.6$ (High Strength)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Density [kg/m$^3$]</td>
<td>2,100</td>
<td>1,400</td>
<td>1,700</td>
</tr>
<tr>
<td>Tensile Modulus (GPa)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Long.</td>
<td>45</td>
<td>76</td>
<td>220</td>
</tr>
<tr>
<td>Trans.</td>
<td>12</td>
<td>5.5</td>
<td>6.9</td>
</tr>
<tr>
<td>Tensile Strength (MPa)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Long.</td>
<td>1,020</td>
<td>1,380</td>
<td>760</td>
</tr>
<tr>
<td>Trans.</td>
<td>40</td>
<td>30</td>
<td>20</td>
</tr>
<tr>
<td>Ultimate Tensile Strain ( )</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Long.</td>
<td>2.3</td>
<td>1.8</td>
<td>0.9</td>
</tr>
<tr>
<td>Trans.</td>
<td>0.4</td>
<td>0.5</td>
<td>0.4</td>
</tr>
<tr>
<td>Cost ($\text{US/kg}$)</td>
<td>Fiber</td>
<td>1.90 - 3.30</td>
<td>31</td>
</tr>
<tr>
<td></td>
<td>Comp.</td>
<td>22</td>
<td>55 - 62</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>200 - 275</td>
</tr>
</tbody>
</table>

### 1.4.4 Polymeric Nanocomposites

A nanocomposite is a composite in which the dispersed material is a particle with at least one dimension no greater than 100 nanometers (nm). The nanoparticles are usually in the shape of a fiber so its diameter is restricted to being smaller than 100 [nm]. Using dispersed material with dimensions on the nanometer scale rather than micrometer scale can result in large improvements in the created composite. Since the diameter of an average polymer coil is 40 [nm], using nanoparticles for the filler can improve the reinforcement. By using a dispersed material that is on the same size scale as the matrix material it ensures that they will better interact and bond with each other [7].

Thanks in part to nanoparticles’ incredibly small size, they inherently have very high surface-to-volume ratios. This means that for each nanoparticle there is a large amount of interaction with the polymers which of course translates to a large amount of reinforcement. Consequently, nanocomposites can provide considerably more reinforcement than an equal amount of the macrocomposite counterpart. Therefore, a lower percentage of nanoparticles is necessary (in the range of 0.5% to 5% by weight) to create the composite [7].

Another reason why nanoparticles prove to be an effective reinforcing material is that
they have a high aspect ratio (their length is considerably larger than their diameter). As previously mentioned, this increases the amount of reinforcement that the particle provides because the maximum load is achieved over a greater segment of the length and thus a greater amount of the load can be transmitted from the polymer matrix to the nanoparticles. In addition to this, since nanoparticles are so small they are less likely to contain flaws and as a result their actual strength is relatively close to their theoretical value [8].

Although nanocomposites appear to be very promising in terms of material properties, they are still very much at the experimental stage. Since the long-term goal of this work is the development of an industrial arm, fiber composites were selected since they are a more mature technology.

1.4.5 Fiber-Polymer Composite Production Processes

The main production processes for fiber-reinforced composites are pultrusion, autoclave forming, resin transfer molding, and filament winding. All three primary fiber materials can be produced using these techniques but there are subtle differences that result in differences in the cost of production. Glass fibers adhere poorly to polymers which require additional steps to be performed in order to produce the composite. As a result, the cost of the composites using the three different fibers is much closer than that of the fibers alone. As indicated in Table 1.2, aramid fiber composites and carbon fiber composites are two and a half and eleven times as expensive as glass fiber composites, respectively [5].

Prepregs are ready made tapes of continuous fiber weaves preimpregnated with a polymer resin that is only partially cured. They enable the user to directly mold and fully cure the product without having to add any resin. They are used in a hand lay-up process in which layers of prepregs are stacked in a specific manner (with fibers in certain directions for each layer) until the desired thickness of the part is achieved.
The temperature and pressure is then elevated to cure it and produce the polymer composite part. Prepregs can also be utilized to simplify some of the manufacturing techniques as the step of drawing the fibers through a resin bath is eliminated.

Pultrusion is a technique in which fibers are drawn through a resin tank to impregnate them before pulling them through a die that preforms them to the desired shape and establishes the resin/fiber ratio. It then passes through a heated curing die that shapes it into the final shape and initiates curing. A pulling device is located at the end to draw the stock through the different stages of the process [5]. Pultrusion is a quick and cost-effective technique for producing components that have a continuous length and a constant cross-sectional shape.

The autoclave forming processing technique is able to produce parts of practically any shape and as a result is a fairly complicated process. First, a peel ply made out of nylon or cellophane is coated with a releasing agent like Teflon and placed on a mold. Prepregs are used to reduce the number of steps involved and are laid up one ply at a time by automated means or by hand. After the lay-up is sealed at the edges to form a vacuum seal, bleeder sheets made of glass cloth are placed on the edges and top of the lay-up in order to get rid of volatiles and excess resin. Vacuum connections are then placed over the bleeder sheets and the lay-up is bagged. A partial vacuum is developed and the entire assembly is placed into an autoclave and heat and pressure are applied to cure the matrix material and form the polymer composite [6].

Resin transfer molding is a relatively simple process. A low viscosity resin is injected under low pressure into a closed mold that contains the fiber preforms. Once all the matrix material has been injected the part is allowed to cure. Beyond being quick, it is also less expensive than hand lay-up and offers the possibility of automation. However, the process is expensive since it requires two molds.

The filament winding technique begins by impregnating fibers by drawing them through an in-line resin bath and winding them around a mandrel. Alternatively, prepregs can
be used instead but are often not used because they do not allow the user to control the properties of the composites and are more expensive to purchase. Depending on the desired properties of the product, different winding patterns like helical, polar, and circumferential are used. After the appropriate number of layers are applied, the part is allowed to cure in an oven or at room temperature. Once fully cured, the mandrel is removed. This technique can be used to create open or closed-ended hollow products. For closed-ended the mandrel needs to be made out of a material that can be broken away in order to remove it from the part. Filament winding is favoured for hollow parts because it is a very quick process since it is automated.

United States (US) patents 2785442, 3429758, and 4878984 [9–11] describe a method or apparatus for the filament winding technique. The first two patents are fairly similar in that they involve winding layers of fibers with opposing angles around a mold [9, 10]. US patent 3429758 however has improvements over US patent 2785442 because it surrounds the layers of fibers with foam and concludes with a final layer of transparent plastic for improved aesthetics and weathering characteristics [10]. US patent 4878984 describes a filament winding machine that utilizes a parallelogram mechanism to apply a plurality of equally spaced parallel fiber slivers. By expanding the parallelogram, the distance between slivers and the angle can be altered which leads to a varying wall thickness. This can be utilized to create a taper [11].

US patents 4118814, 4512835, 5609349, 5238716, and 6367225 [12–16] discuss products that are created using the filament winding technique. US patents 4118814, 4512835, and 5609349 introduce a product that is created by cutting a part that is filament wound [12–14]. US patent 5609349 utilizes a diamond blade circular saw to cut the part and ensure that the strength of the component is not compromised [14]. US patent 5238716 introduces a hollow composite beam with a rectangular shape [15]. It is produced by filament winding an interior layer and then placing four previously made uni-directional fiber plates around the interior layer and then winding an exterior
The advantage of using the pre-made plates is that it enables the beam to be made with a non-uniform thickness in order to improve its compressive and tensile strength.

US patent 6367225, describes a filament wound light pole that is 20 feet long and can support a 300 pound force at its opposite end without exceeding a deflection of 20 inches [16]. In order to successfully do so, the light pole is constructed to include a taper, the tension in the tow thread during the filament winding process is between 30 and 100 pounds, and the pole is made to include multiple layers in which the interior and exterior circumferential layers surround a layer of helical wound fibers.

1.4.6 Structural Composites

A structural composite is typically made up of both homogeneous (a body in which the properties are the same at all points in the body) and composite materials. Its properties not only depend on the properties of the materials that make up the part, but also on the geometry of the parts [5].

A laminar polymer composite is a primary type of structural composite composed of plies of fiber-reinforced composites stacked together. For this reason, prepregs are often used in the construction of the laminate. The plies are stacked together such that the high-strength direction (direction of the fibers) of each successive layer varies. By using this stacking sequence, it ensures that the laminar composite has relatively high strength in a number of directions in the two-dimensional plane. Consequently, the strength in any given direction is less than the maximum that is achieved by having the layers stacked in a uni-directional manner. Additionally, by using different orientations of the layers, different properties can be enhanced. For example a ski, as shown in Figure 1.4a, has the layers oriented in different sequences in order to provide both longitudinal and torsional stiffness. Surrounding these fiber-reinforced composite layers are the homogeneous bodies that are used to enable the ski to perform as desired.
(for example the steel edge acts as a blade that can cut into the snow enabling the skier to turn) [5].

Hybrid laminar composites can also be used to create a composite with a better combination of properties. A composite is considered a hybrid if more than one fiber or one matrix material is used in the laminate. The ply can consist of two different fibers or matrix materials or the laminate can be made up of plies that have different fiber or matrix materials [5].

Sandwich panels are the other primary class of structural composites. As shown in Figure 1.4b, they consist of two outer face sheets that are adhered to and separated by a core. The face sheets are made of a high-strength composite material such as graphite-epoxy composite, whereas the core is constructed of a lightweight material like foam. It is necessary to use a material that provides high strength and stiffness for the faces because it is here that the maximum stresses occur and the majority of the axial forces are supported. Therefore the core does not need to be as strong and rigid so a lightweight material with lower strength and elastic modulus is chosen.

![Figure 1.4: a) Laminar Composite Example - Skies b) Sandwich Panels [5]](image)

Since the resistance to bending of a rectangular cross-sectional plate or beam is proportional to the cube of the thickness, it is optimal to use a core with a larger thickness
and thus using a less dense material can keep the total weight minimal. Additionally, the core’s purpose is to separate the faces and resist deformations perpendicular to the face plane as well as providing shear rigidity along planes that are perpendicular to the face plane. Different shapes like the honeycombs shown in Figure 1.4b can be used to reduce the amount of material and thus the total weight [5].

1.4.7 Methods of Strengthening Hollow Composites

The following patents describe products that use an internal support structure to improve the strength of a hollow component. US patent 1141067 [17] describes a method for reinforcing a tube. An initially hollow tube is filled with a series of identical hollow segments so as to create a hollow tube with an internal support system. Additionally, it is not limited to the number of equally sized segments that are inserted and fixed into the tube.

US patents 3339326 and 3544417 [18,19] describe support structures in which a series of elongated members are arranged to provide support. The interlocking members have a hollow triangular cross-section in order to create a lightweight and strong structure. In addition to the elongated members, foam is added inside of them to improve the strength without significantly adding to the weight. The methods for creating the two support structures differ however in that the elongated member is first made and then filled with foam for the first patent [18], but for the second patent the foam core is first made and then the fiberglass fabric is wrapped around it [19].

US patents 6081955, 3779487, 4223053, and 6655633 [20–23] describe support systems that are made up of a series of elongated members that are attached together to form a support system as illustrated in Figure 1.5. The modular units of these patents are considerably long, in the range of 20 feet. Additionally, the cross-sectional geometries have a variety of shapes ranging from triangular to circular and including variations of these simple shapes. The units are sandwiched between an upper and lower layer
that contacts all of the elongated members. These layers transfer the applied forces to the members that then in turn support the load. Additionally, the modular elongated members are designed to be hollow in order to keep the weight and cost to a minimum while still providing a considerable increase in the strength. US patent 6081955 differs from the rest of the patents in that the modular support structure made of a composite material is produced using hand or automatic lay-up whereas the other three patents describe a support structure that is produced using filament winding. However, all these patents attach the individual modular units by assembling them in the proper orientation and applying a resin and curing it.

Figure 1.5: Series of Elongated Members for Strengthening Support Systems [20]

The modular fiber reinforced bridge described in [24] bonds the individual modular units by first assembling them and then filament winding around the collection of the individual units. This method is beneficial over that described in the above patents because it provides a stronger bond.
1.5 Review of Literature on Composite Robots

Since composite materials are a combination of a reinforcing material and support material their mechanical characteristics can be controlled. Different reinforcing and support materials can be combined to create an endless number of variations with drastically different mechanical properties. Additionally, the percentages of the two materials can be varied to offer further range of the mechanical properties. Furthermore, the way in which the two materials interact can be varied to further alter the properties of the composite material.

The high degree of control associated with composite materials allows the user to construct a material that is specifically designed for their application. Therefore, composite materials are desirable for the construction of robotic arms because they have a very high potential for improvements in stiffness, strength, and damping capability. This is why there has been a considerable amount of research performed on the production of robotic arms out of polymer composites over the previous 20 years.

1.5.1 Production Techniques

Choi et al. [25] constructed a single link arm using an AS4/3501-5A magnamite graphite composite. It was produced using 20 laminate plies. All the plies were oriented in the same direction, parallel to the longitudinal axis ($0^\circ$) to increase the bending stiffness of the link. The link was fabricated using the hand lay-up technique using prepreg tape. Vacuuming and autoclaving were used to facilitate the production process by shaping the surfaces of the panel and applying a pressure to evenly apply the epoxy resin and facilitate the curing process.

In 2001, Lee et al. [26] also created a polymer composite single link arm that was modeled as a cantilever beam. The link was created as a hollow cylindrical tube with a length of 564 [mm] and diameter of 110 [mm]. They constructed the link using 66
plies of USN125 carbon prepreg to give the link a total thickness of 7 [mm]. They used a ± 5° stacking angle since 0° could induce axial splitting under vertical loads due to its low transverse stiffness and strength. The plies were laid-up on the mandrel by hand and the link was manufactured using the autoclave vacuum bag process.

Lee et al. [27] also manufactured a SCARA (selectively compliant assembly robot arm) type composite robot for handling large LCD glass panels as shown in Figure 1.6. The end-effector was produced using USN125 carbon-epoxy composite. The hollow box-beam end-effector was created using prepreg hand lay-up of 16 plies with ± 15° fiber orientation. The plies were laid directly onto a Nomex honeycomb (HRH-10-1/8-4.0) core that was used to improve the rigidity of the arm without adding any significant increase in weight. The entire assembly was cured using the autoclave vacuum bag degassing method.

Figure 1.6: Carbon-Epoxy Composite SCARA robot [27]

Lee et al. [28] subsequently redesigned the links of the SCARA type robot. All the links were still designed to have a box-beam shape; however, a polyurethane foam core was used. The prepreg plies were hand-laid at an angle of ±30° on the closed mold and the foam injected between the layers. The mold was then placed in an autoclave to cure both the epoxy and the foam. The foam core adhered to the composite as the excess epoxy resin penetrated the foam core.
Lessard et al. [29] integrated a polymer composite link into a 7-DOF manipulator. They used LTM25 graphite with epoxy for the construction of the second link (the upper-arm). They designed the link to be a box-beam (hollow rectangular cross-section) with a pair of holes on each end. FEA determined that the optimal laminate lay-up is a repeating pattern of $[0^\circ/0^\circ/45^\circ/-45^\circ]$. Since the force of the actuators is exerted on the holes, two woven patches that include a $0^\circ$ layer and $90^\circ$ layer and another two woven patches that include a $45^\circ$ and $-45^\circ$ layers were added to the hole region to improve the strength at this location.

A stainless steel end-fitting is attached to the link such that it surrounds the exterior profile of the link. An external aluminum mold was used for the construction to control the external dimensions of the link. The plies were cut from uni-directional prepreg and hand-laid. The authors investigated sandwiching both an aluminum flex-core and Nomex honeycomb between the layers but it was determined that they hindered the compaction of the layers around the corners. Additionally, it was determined that the cores barely improved the in-plane strength around the holes so they opted not to include any sandwich support. Additionally, the part was cured at a maximum temperature of 79 °C using a vacuum-bagging technique.

### 1.5.2 Connection Techniques

As previously mentioned, the link discussed in [29] used an end-fitting that surrounded the exterior of the link to facilitate joining. The end-fitting was bonded to the composite link using Hysol EA 9430 rather than being bolted because it provides a stronger connection and does not diminish the structural integrity as a result of drilling. Graphite is quite cathodic and as a consequence can cause metals to corrode. Additionally, it has a very low coefficient of thermal expansion. Therefore the authors chose to construct the end-fitting out of stainless steel to help prevent corrosion and reduce thermal stress. The end-fitting was also constructed with a $7^\circ$ chamfer angle
to help reduce the stress-concentration at the edge of the bond. As for the link discussed in [26], a steel flange was used to facilitate joining of the composite arm to an electrical discharge wire cutting machine. In this case, the steel flange was inserted into the arm as illustrated in Figure 1.7. To accommodate bonding of the flange to the composite tube, the flange was segmented. Accordingly, it used both an interference fit as well as an adhesive epoxy (IPCO 9923). FEA revealed that the optimal length of the interference fitting area and adhesive bonding area of the flange were 32 [mm] and 28 [mm], respectively. To help reduce the hoop stress in the joint part under vertical loads, six reinforcing plies of 90° prepregs were applied on the exterior of the composite arm at the joint locations.

![Figure 1.7: Schematic of Attachment of Steel Flange to Composite Arm [26]](image)

1.5.3 Results Achieved

Many physical and computational tests have been performed on robotic systems constructed of polymer composites and the more traditional steel and aluminum arms. The results showed considerable improvement in both dynamic and static properties.
of the robotic system made from composites. The composite arm that is described in [25] was designed to have a flexural rigidity equal to that of an aluminum counterpart. In doing so, it too was designed as a solid rectangular beam with a length of 812 [mm] and a width of 19.02 [mm], but offered a reduction in the thickness from 3.14 [mm] to 2.50 [mm]. This equates to a reduction in the weight of over 50%, from 0.133 [kg] to 0.060 [kg]. With the use of a proportional and derivative (PD) controller, both arms were tested. It was found that the composite arm reduced the maximum overshoot from 1.02° to 0.3°, reduced the energy consumed by the motor used to run the experiment by 40%, and decreased the maximum deflection at the free end of the link from 3.4 [mm] to 1.7 [mm], all the while reducing the settling time.

Krishnamurthy et al. [30] performed computational tests on a similar link using an extended Hamilton’s principle to derive the equation of motion and studied the unconstrained and constrained mode of vibration. They determined that the motion-induced tip displacement as well as the torque required for the manoeuvre was considerably less for the links constructed of graphite-epoxy and boron-epoxy composite than that of aluminum and steel counterparts. They also determined that composite links are preferred because the magnitude of the control spillover effects is very small, which makes it easier for designing control systems.

As for the single link carbon-epoxy composite arm created by Lee et al. [26], it reduced the weight of its cast iron predecessor by 58%, from 26.5 [kg] to 11.2 [kg]. The equally-sized arms had close to the same deflection when a 10.2 [kg] payload was applied at the end of the link. The cast iron arm deflected 38 [µm] and the composite arm deflected 40 [µm]. While essentially offering the same stiffness with an impressive reduction in weight, the composite arm also offered improvements in the dynamic characteristics. Under fixed-free conditions, the composite arm had a fundamental natural frequency that is 1.41 times higher and a damping ratio that is 1.20 times greater than the cast
iron arm. Under free-free conditions, the fundamental natural frequency and damping ratio of the composite arm was 1.27 and 5.00 times greater than that of the cast iron arm.

The graphite-epoxy composite link that was created by Lessard et al. [29] also showed improvements in the natural frequency. The link in question of the 7-DOF manipulator was analyzed by itself as a cantilever arm. With maximum torque applied by the actuators, a vertical force of 3,790 [N] acting downwards and a horizontal force of 1,037 [N] acting to the right, was applied at the end of the link. This resulted in a deflection of 0.38 [mm] for the composite link compared to 0.35 [mm] and 0.11 [mm] for its aluminum and steel counterparts. Although the composite arm had the largest amount of deflection, it weighed only 418 [g], considerably less than the 729 [g] for the aluminum arm and the 2,133 [g] for the steel arm of identical dimensions. The vertical natural frequency of the composite link was around three times greater than that of the aluminum and steel links. The improvements in the horizontal and extensional natural frequency were even better, being approximately four times greater.

Caprino and Langella [31] also investigated the fundamental frequency of composite arms. They created equations for a high modulus graphite-epoxy composite hollow cylindrical link modeled as a cantilever beam with a concentrated load acting on the end. They determined that the ratio of the length to the radius, \( l/R \), for the link had a dramatic effect on the fundamental frequency. For a \( \pm \theta^\circ \) layering sequence, they determined that the optimal fiber angle \( \theta \) was decreased as the \( l/R \) ratio was increased, indicating that longer beams require a fiber angle close to 0\(^\circ\). For shorter links the fundamental frequency was maximized with a fiber angle closer to 45\(^\circ\) since shear is the primary force acting on the link. Similarly, for a \((0/\pm 45^\circ_n)\) repeating pattern, where \( n \) is a percentage of 0\(^\circ\) layers, a higher \( l/R \) ratio requires a lower percentage of \( \pm 45^\circ \) layers for maximum fundamental frequency as shown in Figure 1.8.
Oral and Ider [32] created dynamic equations that were capable of determining the deviations from the desired path for a three revolute joint robotic arm with filament-wound tapered box-beam links as illustrated in Figure 1.9. A robot with aluminum links and one with composite links of equal length and weight were tested. The results indicated that for a translation in one direction requiring rotation of both links, the deviation in all directions was less for the robot with composite links than it was for the robot with aluminum links. In fact, the total accuracy was improved by about four times.

Ghazavi and Gordaninejad [33] also performed computational analysis on a $R \perp R \parallel R$ robot where $R$ denotes a revolute joint, $\perp$ denotes two joints being perpendicular to each other, and $\parallel$ denotes two joints being parallel to each other. They studied a solid graphite-epoxy composite third link that consists of 8 plies with a repeating layering sequence of $[0^\circ/-45^\circ/45^\circ/90^\circ]$. This pattern was chosen to maintain a good balance between flexural and shear stiffness. The composite arm was approximately half the weight of the aluminum link of equal dimensions. This led to a 15%, 27%, and 28% peak torque reduction for the first, second, and third joints, respectively.
The composite link also had improved steady-state error of the tip manipulator and a faster and less oscillatory response. For similar reasons, the composite arm offered reductions in the forces over its aluminum counterpart. Additionally, the axial and out-of-plane transverse deflections of the composite link were 47% and 25% less than that for the aluminum arm.

![R ⊥ R ∥ R Robotic Arm](image)

Figure 1.9: $R \perp R \parallel R$ Robotic Arm [32]

The carbon-epoxy composite end-effector for a SCARA type robot that handles LCD glass panels as described in Section 1.5.1 showed considerable improvements over its aluminum predecessor [27]. The composite part weighed only 2.19 [kg], less than half of the 4.44 [kg] aluminum model. Consequently, the total deflection of the composite end-effector when loaded with an LCD screen was only 2.52 [mm] whereas the aluminum end effector deflected a total of 4.87 [mm]. In addition to this, the composite end-effector had better dynamic characteristics than the aluminum counterpart. The fundamental natural frequency and damping ratio of the composite end-effector were 1.5 and 3.6 times greater than those of the aluminum end-effector.
1.5.4 Composite Arms Used in Industry

German company Schubert [34] redesigned their two main-arm pick-and-place robots to have hollow carbon fiber reinforced plastic (CFRP) links. These links were implemented to replace the old aluminum links in order to meet rising performance requirements. The links were designed to include a taper such that the cross-section of the link was larger on the end closer to the base in order to reduce the bending moment. Laminate inserts and radial stiffeners were also included at the joint location. The link was constructed with a combination of prepregs with fiber directions of $0^\circ$, $45^\circ$, and $90^\circ$ to provide bending, shear, and torsional strength. The new CFRP links provided a total weight reduction of 25% over the aluminum predecessor. Additionally, the maximum distortion inside the hollow link was measured at 0.25 [mm]. In addition to improving the stiffness of the robot, an improvement in the dynamic behaviour of 50% was achieved. These improvements lead to the replacement of the aluminum model in the beginning of 2007.

CBW Automation [35] currently offers a variety of robots that are constructed of carbon fiber composites. They, however, are quite different compared to the robots mentioned in the previous sections. Their robots all operate via a gantry system. Additionally, the robots have very limited mobility, with the majority of them only having 1-DOF. These robots are used for transporting large heavy parts. The use of carbon fiber composite materials over traditional materials has lead to significantly shorter cycle times.

In 2005, Kuka [36] began selling the first commercially available large-scale robotic arm that features the use of composite materials. The third link of the arm is constructed of carbon fiber composite. The arm is capable of stacking loads weighing up to 100 [kg] to heights of up to 3 [m], at rates as high as 600 palletizing cycles per hour.

The robotic arm weighs 1,200 [kg] compared to a similar traditional Kuka robotic arm that weighs 1,500 [kg]. This arm however has a reach of only 3.2 [m] whereas the
traditional arm has a reach of 3.5 [m], however, the weight reduction indicates good improvement over its traditional counterpart [1].

Figure 1.10: Kuka Robotic Arm Featuring Composite Third Link [1]

1.5.5 Opportunities

Although much research has been performed on composite robotic arms and a few commercial arms featuring the use of composite materials are available, an opportunity exists to investigate novel cross-sectional geometries to improve the performance of composite robotic arms. Most composite robot arms feature hollow, tubular structures. Although these structures are strong, their strength could potentially be improved by adding an internal support structure.

1.6 Summary of Contents

The contents of the remainder of this thesis are as follows. In Chapter 2, the configuration of the industrial-sized 6-DOF robotic arm is selected. The cross-section of the link is studied using both FEA and optimization. Using the information and
results obtained from these preliminary investigations, the links for a main-arm proof-of-concept prototype are designed and developed as outlined in Chapter 3. Chapter 4 discusses the different tests that were performed on the proof-of-concept prototype and outlines its performance. A FEA technique for modeling the proof-of-concept prototype link as an anisotropic material is also presented. The results from the tests of the physical and virtual prototype are also compared. The results of Chapter 4 are used to design the full-scale industrial-sized robotic arm in Chapter 5. The thesis is concluded with Chapter 6 and suggestions for future work required before the industrial-sized arm can be manufactured.
Chapter 2

Link Design

This chapter outlines the preliminary design decisions for the construction of the polymer composite robotic arm. The main configuration for the 6-DOF robotic arm was selected. An optimal layout for the robotic arm was also selected. FEA was performed on a variety of links with various cross-sections. An ideal cross-section was selected based on minimizing the deflection relative to the weight of the link. Finally, the dimensions of the link for a 2-DOF proof-of-concept prototype were optimized for minimal deflection with the use of MATLAB.

2.1 Manipulator Layout Selection

2.1.1 Comparison of Various Types of Robotic Arms

Before beginning to design the robotic arm to be used for large-scale applications, the type synthesis of the robotic arm must be performed. The robotic arm can either be serial, parallel, or a hybrid of the two. As illustrated in Figure 2.1, a serial configuration is one in which the links are connected to each other in series, one after another, forming an open loop mechanism. A robotic arm with a parallel configuration

\footnote{Please see [37] for paper on this topic}
consists of a mobile platform that is connected to the base by at least two independent kinematic chains that form a closed loop. A hybrid robot is a combination of the two configurations. Consequently, a hybrid robot can either have a serial configuration at its base and a parallel base connected afterwards, or have a parallel configuration at its base and conclude with a serial configuration.

Figure 2.1: Schematics of Different Types of Main Configurations for Robotic Arms
Serial and parallel configurations result in robotic arms that have many varying characteristics. A serial robotic arm is distinguished by having a large workspace. A robotic arm with a serial configuration can reach over a significantly larger area and or volume than a robotic arm that has the same size links using a parallel configuration. In addition to this, parallel robotic arms generally cannot reach around obstacles like their serial counterparts can.

A robotic arm with a serial configuration is considerably simpler than one with a parallel configuration. Since one link provides one motion rather than a group of links it is less complex. Consequently, a robotic arm with a serial configuration is relatively easy to design and manufacture.

For parallel robots, since multiple limbs are required, they are significantly more complex than serial robotic arms. As a result, a parallel robotic arm costs a lot more to produce than a serial one. Another result of the increased complexity of a parallel robotic arm is that in comparison to a serial robotic arm it is difficult to control and operate. The kinematics of a parallel robotic arm are more complex than that of a serial arm which makes the algorithms used to provide motion for parallel arms considerably more complicated than those needed for serial robotic arms.

Robotic arms with a parallel configuration have a high stiffness since the end-effector is supported at several locations (whereas for a serial robotic arm it is only supported at one location). Furthermore, since many links are connected in a closed chain manner, the errors and displacements are averaged over the number of links. This is not the case for a serial robotic arm, where the errors are additive being the sum of all axis errors of the links. Consequently, a parallel configuration results in a robotic arm with greater accuracy than a serial configuration. A parallel configuration usually results in a robotic arm with a lower weight than a serial robotic arm not only because the improved accuracy allows for the possibility of using smaller links, but in the parallel configuration the motors can be mounted on the base. As a result, the links can be
designed to be smaller and thus weigh less since they do not have to support the weight of the motors. However, a robotic arm with a parallel configuration is known to lose stiffness in singular positions. In fact, this loss can be so severe that it causes it to shake or even become free to move despite all the joints being locked. Therefore, special care needs to be taken to avoid such scenarios.

A hybrid robotic arm could be very beneficial for this application because it allows for the possibility of constructing an arm that has a large workspace and high stiffness while keeping the weight to a minimum. However, the complexity of a hybrid results in a substantial increase in the cost of production of the arm and the control of the system is more difficult.

Based on the above, a serial layout was selected due to its inherent simplicity in terms of control and construction.

### 2.1.2 Serial Arm Configuration

There are numerous combinations of six joints that could be used to achieve 6-DOF motion. However, a kinematically-simple manipulator layout is ideal because it incorporates a spherical group of joints at the wrist connected to the end of a main-arm comprised of three successfully parallel or perpendicular joints with no unnecessary offsets or link lengths between joints [38]. As a result, it is easier to develop the kinematics of the arm for this type of layout opposed to one that is not kinematically simple. The use of a kinematically-simple 6-DOF robotic arm reduces the difficulty involved in controlling and operating the arm. Additionally, the simplicity of the layout makes it easier to design and manufacture. Since unnecessary offsets and link lengths are avoided for this type of layout, the amount of material for the robotic arm is kept to a minimum. Consequently, this helps to improve the payload-to-weight ratio.
There are four groups of kinematically-simple main-arm layouts, those with three revolute joints, two revolute joints and one prismatic joint, two prismatic joints and one revolute joint, and three prismatic joints. Prismatic joints add significant weight to the robot over revolute joints and thus to design a robotic arm with the highest payload-to-weight ratio, a layout with three revolute joints is to be utilized. There are five different kinematically-simple main-arm layouts that are comprised of three revolute joints. The $R \perp R \parallel R$ manipulator layout as shown in Figure 2.2 was chosen.

The $R \perp R \parallel R$ layout offers the largest spherical workspace of the five layouts with three revolute joints. Since the robotic arm that is being designed is to operate on large surfaces with highly complex curvature, this layout allows the end-effector to reach the greatest amount of surface area without moving the entire robotic manipulator. In order for a robotic arm with one of the other four joint main-arm layouts to be able to reach an equivalent amount of surface area, the links must be longer.

The $R \perp R \parallel R$ layout is also preferred because the two main links (links A and B in Figure 2.2) of the robotic arm work in the same plane. This is beneficial since it helps to keep the torsion to a minimum. It has been shown that for a SCARA robot the amount of deflection experienced by the end-effector is greatest when there is a combination of both bending and torsion [39]. Consequently, this feature helps to minimize the end-effector deflection.

Since the $R \perp R \parallel R$ layout has the second and third joints parallel, it offers the possibility of the motors of the main arm being mounted as close as possible to the base by utilizing a simple drive-train for movement of the third joint. By reducing the amount of weight that is located between the two main links both the amount of stress and displacement in the links is reduced and consequently high accuracy and repeatability can be achieved while using less material.
2.2 Analysis of the Cross-Section of the Two Main Links

2.2.1 Cross-Section Testing Procedure

As defined by the specifications, the industrial robotic arm is to have a total reach of 5 [m]. The two main links are the links that occur after the second and third joint as illustrated in Figure 2.2. Since the two main links supply the majority of the reach of the kinematically-simple 6-DOF robotic arm, for the purpose of this preliminary study the two main links are defined as having a total length of 5 [m]. The analysis was performed under the worst possible scenario, fully extended at its maximum reach. In this position the two main links of the arm are parallel and thus they can be analyzed as a single straight member with a constant cross-sectional geometry. For the purposes of testing the different cross-sectional geometries, the strength and stiffness of the member representing the two main links was examined.

In keeping with achieving a reduction in the overall weight of the arm, it is imperative that the links be designed such that the amount of material is kept to a minimum. In order to do so, the two main links of the robotic arm should be designed as thin-walled
members. To satisfy the thin-walled beam criterion, the thickness must be at least 10 times smaller than any characteristic dimension of the cross-section, in this case the radius. The radius must also not be more than one tenth the size of the length [40]. Therefore, a thickness of 10 [mm] and a radius of 127 [mm] (5 [in]) were chosen. Since the reach of the full-scale robotic arm is 5 [m], then a length of 2.5 [m] for each of the main links is long enough that a radius of 127 [mm] (5 [in]) satisfies the rules for a thin-walled beam.

For this preliminary analysis, only the effect of using different cross-sectional geometries on the stiffness of the link is of concern. Therefore, the aforementioned dimensions for the thickness of the walls, diameter of the exterior profile, and the length of the links are held constant.

The links of the robotic arm are constructed of high-modulus carbon fiber-epoxy composite with a 60% fiber volume. It was chosen because it offers the greatest stiffness due to its high tensile modulus and low ultimate tensile strain as indicated in Table 1.2 of Section 1.4.3.2. However, for this analysis, it was modeled as an isotropic material with a density of 1,700 [kg/m$^3$], modulus of elasticity of 220 [GPa], and Poisson’s ratio of 0.25 [5]. Since the purpose of this study was solely to compare different cross-sectional geometries, modeling it as a simplified isotropic material still provides sufficient information on how varying the cross-sectional shape reduces the amount of deflection the end-effector experiences.

In order to properly analyze the stiffness and strength of the links with various cross-sections, a force that represented the payload of the robotic arm, as well as the weight of the arm itself was applied. A distributive force acting downwards that represented the weight of the two main links was applied evenly along the length of the single straight member. Additionally, a force $F$ measured in Newtons (N) acting in the same direction was applied at the end of the arm. It was calculated using:
where $m_{\text{payload}}$ is the mass of the payload, $m_{\text{wrist}}$ is the mass of the end-effector, links, and motors that make up the group of spherical joints at the wrist (both measured in [kg]), and $g$ is the acceleration due to gravity, 9.81 [m/s$^2$]. By using the specified payload of 50 [kg] and 30 [kg] as the assumed weight of the wrist components of the robotic arm, the force that was applied at the end of the member was found to be 784.8 [N].

This force was applied on the last 200 [mm] of the upper portion of the arm as demonstrated in Figure 2.3. The force was applied on this area as it sufficiently represents the area of the link that would be supporting the wrist components and thus the load that is being held by the gripper. The other end of the arm was fixed in all directions such that it offered 0-DOF so as to represent the arm being locked in the fully extended position in order to determine the effectiveness of the cross-sectional geometry in terms of stiffness and strength when the bending force is at its maximum.

Figure 2.3: Force Applied on Single Straight Member Representing Two Main Links
For the purpose of analyzing the various cross-sectional geometries, FEA was conducted using UGS NX 5.0 with NX Nastran as the solver by applying the aforementioned forces. The thin-walled beam representing the robotic arm fully extended was analyzed using the Structural analysis type. Finally, the solution type chosen for the FEA procedure was SESTATIC 101 - single constraint. The robotic arm was represented as a mesh of 3D 10-noded tetrahedral elements in order to perform the FEA.

### 2.2.2 Results of Cross-Section Analysis for the Main Links

Different families of cross-sections for the two main links were tested using the procedure described in the previous section. The different families of cross-sections and the stiffness and strength of the single straight member which represents the two main links extended at their maximum length are discussed below.

#### 2.2.2.1 Basic Shaped Hollow Tubes

The initial types of link geometry that were tested were hollow. Three simple basic shapes were chosen for the cross-section: circular, square, and rectangular (as shown in Figure 2.4). To effectively compare the three different cross-sectional geometries, the dimensions of the tubes were chosen such that their perimeters were nearly equal, which results in a similar volume and mass. As a result, using a constant wall thickness of 10 [mm], the two main links of the robotic arm have a similar volume and thus mass for the different geometries of this first family of cross-sections. As previously mentioned, the diameter of the circular tube representing the two main links fully extended was chosen to be 254 [mm] (10 [in]). This equates to a perimeter of 797.96 [mm]. Therefore, the dimensions of the other two tubes were chosen such that their perimeters were equal to 798 [mm]. Consequently, the square tube that was analyzed has four sides that are 199.5 [mm] long. Since the serial robot layout that was chosen
ensures that the main two links always work in a vertical frame, using links with a larger height than width ensures that it provides greater support for the payload that is acting downwards due to the force of gravity. In order to provide more support, the tube with the rectangular cross-section was designed such that its height was equal to two times its width. Therefore, the height and width of the rectangular tube were 266 [mm] and 133 [mm], respectively.

![Cross-Sections of Family of Basic Shaped Hollow Tubes](image)

Figure 2.4: Cross-Sections of Family of Basic Shaped Hollow Tubes

The procedure that is described in the previous section was used to determine the maximum amount of displacement and stress in the member representing the two main links when fully extended for the first family of cross-sectional geometries. It was determined that the maximum displacement and stress in the hollow tube with the circular cross-section were 3.333 [mm] and 13.72 [MPa]. For the square hollow tube, the same test indicated that the maximum displacement and stress experienced in the member were 4.169 [mm] and 14.31 [MPa]. As expected, the hollow tube with the rectangular cross-section showed improvements over the square tube having a maximum displacement of 2.775 [mm] and a maximum stress of 12.18 [MPa]. These results indicated that the rectangular cross-section is favoured over square and circular as it produces the stiffest and strongest links for these loading conditions. However, due to the curvature of the circular tube it would present fewer problems during link manufacture if using a filament winding machine. Therefore, alterations of both the rectangular tube and circular tube were investigated.
2.2.2.2 Circular Tubes with Support Ribs

The hollow circular tube was modified by adding support ribs that initiated in the middle of the tube propagating outwards until they connected to the inner wall of the tube. Both an X-support tube and Tri-support tube were tested. The X-support tube has four equally spaced ribs (that form an X) and was tested in two different orientations, while the Tri-support tube has three support ribs and was tested in its three primary orientations as shown in Figure 2.5.

![Figure 2.5: Cross-Sections of Family of Circular Tubes with Support Ribs](image)

For the X-supported tube it was found that the maximum displacement and stress were 3.222 [mm] and 15.02 [MPa] for when the tube was in the orientation illustrated in Design 4 of Figure 2.5 such that the force of the payload was acting at an equal angle to all the ribs. The tests indicated that the results were the same for the member when it was oriented such that the force of the payload was acting in a direction parallel to two ribs as demonstrated in Design 5.

As for the Tri-supported circular tube, the maximum displacement and stress in the member while oriented such that the force of the payload was acting in a direction perpendicular to one of the ribs as shown in Design 6 were determined to be 3.254 [mm] and 13.40 [MPa]. However, with the Tri-supported circular tube in an orientation such that the force of the payload was acting in a direction parallel to one of the ribs (in either of the orientations illustrated in Design 7 and 8) the maximum displacement and stress were found to be 3.303 [mm] and 15.14 [MPa].

Despite the fact that the addition of four ribs (X-supported) rather than three (Tri-
supported) increases the weight of the member, it underwent a lesser amount of displacement as illustrated in Figure 2.9. Additionally, the percentage decrease in the displacement versus the percentage increase in the mass of the two main links in comparison to the hollow circular tube was greater for the X-supported circular tube than the Tri-supported one. This indicates that the use of four support ribs is favoured over three as its results in a reduction in the displacement that is greater in comparison to the increase in the total mass. However, even for the X-supported circular tube the decrease in the maximum displacement was small in comparison to the increase in the mass of the two main links. This small decrease in the maximum displacement does not justify the significant increase in the mass (the percentage increase in the mass of the two main links was more than 10 times the percentage decrease in the deflection). Additionally, the support ribs further complicate the production as a different process needs to be used to produce the support structure since it cannot be produced simultaneously with the outer tube using the filament winding process. All in all, the slight improvements in the stiffness that the addition of the support ribs provide does not justify their use with an outer circular cross-section.

### 2.2.2.3 Circular Tube with Different Shaped Inner Tubes

The cross-section of the circular tube was further modified by adding an inner tube to the support system such that the support ribs connect the exterior of the inner tube to the interior of the exterior tube. The straight member representing the two main links fully extended was tested with a circular, rectangular, square, and diamond inner tube as illustrated in Figure 2.6. The inner circular tube that was tested had an inner diameter of 152.4 [mm] (6 [in]). The other shapes of the inner tubes (rectangular, square and diamond) were designed with a perimeter equal to that of the 152.4 [mm] diameter inner circular tube. The weight of the link varies significantly for the different inner tubes (as can be seen in Figure 2.9) because the ribs required to connect the
inner tube with the outer circular tube vary significantly in length for the differently shaped inner tubes.

![Cross-Sections of Family of Circular Tube with Different Shaped Inner Tubes](image)

For the circular inner tube with three support ribs, the maximum displacement and stress in the member was found to be 2.900 [mm] and 12.74 [MPa]. As expected, the use of four support ribs for the inner circle was found to be an improvement over three ribs, resulted in a maximum displacement and stress of 2.876 [mm] and 12.39 [MPa]. Therefore, the other shaped inner tubes were connected to the outer circular tube by four supporting ribs.

For the inner square tube cross-section, the maximum displacement and stress were found to be 2.984 [mm] and 12.71 [MPa]. For the inner diamond tube as illustrated in Design 12, the maximum displacement and stress were determined to be 2.933 [mm] and 12.99 [MPa]. Similar to the case for the basic shapes, the rectangular inner tube was found to reduce the displacement the most. In fact, the addition of the rectangular inner tube reduced the displacement from 3.333 [mm] for the hollow circular tube to 2.701 [mm], as well it reduced the maximum stress from 13.72 [MPa] to 11.75 [MPa]. However, this design nearly doubled the mass of the two main links, increasing it from 65.16 [kg] to 126.55 [kg]. This means that the percentage increase in mass was around 5 times the percentage decrease in displacement. Therefore, the addition of the inner tube to the support system results in an improvement of the displacement with respect to the increase in mass by more than two times over the family of circular tubes with support ribs discussed above. However, it makes the production process
considerably more complicated. The filament winding process needs to be performed twice, and the support system needs to be connected to both the inner and outer tubes (either by hand lay-up or some mechanical process; either way this drastically increases the time and cost of production).

### 2.2.2.4 Annulus with Different Sized Inner Tubes

Another modification to the circular tube that was investigated was an annulus (a circular tube within a circular tube) with a varying diameter for the inner tube. As was the case for the other investigations, this design was tested using both three and four support ribs. The annulus was tested with inner diameters of 63.5 [mm] (2.5 [in]), 101.6 [mm] (4 [in]), 152.4 [mm] (6 [in]), and 203.2 [mm] (8 [in]) as illustrated in Figure 2.7 (shown with four support ribs).

![Figure 2.7: Cross-Sections of Family of Annulus with Different Sized Inner Tubes](image)

For an inner diameter of 63.5 [mm], the Tri-support Annulus had a maximum displacement and stress of 3.358 [mm] and 14.38 [MPa], whereas the X-support Annulus was found to lower the maximum displacement and stress to 3.266 [mm] and 13.96 [MPa]. For the 101.6 [mm] inner diameter annulus, the maximum displacement and stress for the Tri-support and X-support Annulus were 3.234 [mm], 3.181 [mm], 14.17 [MPa], and 13.68 [MPa], respectively. The results for the annulus with an inner diameter of 152.4 [mm] for both three and four support ribs are mentioned above in Section 2.2.2.1. For an annulus with an inner diameter of 203.2 [mm], the maximum displacement and stress for the Tri-support and X-support Annulus were determined...
to be 2.405 [mm], 2.387 [mm], 10.78 [MPa], and 10.62 [MPa], respectively.

As was the case for the previous tests, the X-support proved to provide better stiffness and strength than the Tri-support. However, as the diameter of the inner tube was increased the improvements that the Tri-support Annulus provides grew closer to those of the X-support. This makes sense because as the inner tube grows the ribs become shorter and thus the amount of support that the ribs provide is lowered. Associated with this, is the fact that the mass of the member for the Tri-supported Annulus also gets closer to that of the X-supported Annulus as the size of the inner diameter is increased (as demonstrated in Figure 2.9).

The results of this study indicated that as the size of the inner tube increased, the strength and stiffness of the member were also improved. In fact, the degree of decrease in displacement with respect to increase in mass increased significantly as the inner diameter grew as illustrated in Figure 2.9. It continues to increase until the inner diameter has grown so much that it is now a hollow tube with a thickness of 20 [mm] (double that of the original case). As previously mentioned, the production of the two main links with an annulus design is more complex and thus it is not worth using when even better results can be achieved with a simple hollow tube.

2.2.2.5 Rectangular Tubes with Support Ribs

The rectangular tube was modified by adding support ribs as illustrated in Figure 2.8. For Design 18, a vertical support in the middle of the rectangle was used. For this I-beam supported rectangular tube, the maximum displacement and stress in the member were determined to be 2.526 [mm] and 11.67 [MPa]. The next support that was tested was a T-support as shown in Design 19 of Figure 2.8. The rectangular tube with this type of support had a maximum displacement and stress of 2.601 [mm] and 12.14 [MPa]. The I-beam support proved to provide more stiffening and strengthening than the T-support because the addition of the horizontal support rib
provided minimal restraint to the force of the payload acting downwards in the vertical direction. As a result, the addition of the mass of the horizontal rib increased the displacement and stress within the member.

![Cross-Sections of Family of Rectangular Tubes with Support Ribs](image)

**Figure 2.8: Cross-Sections of Family of Rectangular Tubes with Support Ribs**

The final support tested for the rectangular tube was an X-support (Design 20). This support system resulted in a maximum displacement and stress of 2.346 [mm] and 10.67 [MPa]. These results indicate that the I-beam support and X-support designs are considerably better than the T-support. Figure 2.9 shows that these two designs offer a percentage reduction of the displacement that is between a third and a quarter of the percentage increase in the mass. Further inspection of Figure 2.9 indicates that the percentage decrease in the displacement in comparison to the percentage increase in the mass that the I-beam support provides is better than that of the X-support. The X-support provides a greater amount of torsional rigidity than the I-beam support and even though the main links operate in a vertical plane, the center of gravity of the end-effector may be offset horizontally thus resulting in torsion in the two main links in which the X-supported rectangular tube is better suited for. The slight improvements in the stiffness and strength that an inner support tube provides does not warrant their use since it makes the production of the links more difficult.
2.2.3 Analysis of Tapering the Main Links

Observation of Figure 2.9 indicates that the I-beam supported and X-supported rectangular tubes (Designs 18 and 20) provide the greatest amount of stiffness relative to the mass of the two main links making them the optimal cross-section geometries for these loading conditions. Therefore, these two link designs were tested by tapering the link in the direction of its length. The dimensions of the cross-section at the end of the member representing the two main links were held constant but the dimensions of the cross-section at the beginning of the member were larger according to the taper angle used as demonstrated in Figure 2.10. For this analysis, the member representing the two main links was tapered with an angle between zero and one degree. Since the links are considerably long, a taper angle of just one degree results in the larger end having a height of 440.55 [mm] and width of 220.27 [mm] compared to 266 [mm] and 133 [mm] for the end of the member. This results in an increase in the mass by
over one third of its original weight and thus using any taper angle beyond this is excessive.

Figure 2.10: Tapered Member Representing the Two Main Links

The member with the two different cross-sectional geometries was analyzed using taper angles of $0.05^\circ$, $0.10^\circ$, $0.15^\circ$, $0.25^\circ$, $0.50^\circ$, and $1.00^\circ$. Table 2.1 lists the results for the maximum displacement and stress within the member at the different angles. The amount of displacement and stress decreases as the taper angle is increased.

Table 2.1: Effect of Taper Angle on the Displacement and Stress

<table>
<thead>
<tr>
<th>Taper Angle</th>
<th>I-beam Supported Displacement</th>
<th>Rectangular Tube Stress</th>
<th>X-Supported Displacement</th>
<th>Rectangular Tube Stress</th>
</tr>
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<tbody>
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<td>2.526 mm</td>
<td>11.67 MPa</td>
<td>2.346 mm</td>
<td>10.67 MPa</td>
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<td>10.60 MPa</td>
<td>2.173 mm</td>
<td>9.971 MPa</td>
</tr>
<tr>
<td>$0.10^\circ$</td>
<td>2.176 mm</td>
<td>9.959 MPa</td>
<td>2.018 mm</td>
<td>9.685 MPa</td>
</tr>
<tr>
<td>$0.15^\circ$</td>
<td>2.030 mm</td>
<td>9.409 MPa</td>
<td>1.878 mm</td>
<td>9.107 MPa</td>
</tr>
<tr>
<td>$0.25^\circ$</td>
<td>1.773 mm</td>
<td>8.736 MPa</td>
<td>1.638 mm</td>
<td>8.197 MPa</td>
</tr>
<tr>
<td>$0.50^\circ$</td>
<td>1.305 mm</td>
<td>6.697 MPa</td>
<td>1.201 mm</td>
<td>6.379 MPa</td>
</tr>
<tr>
<td>$1.00^\circ$</td>
<td>0.782 mm</td>
<td>4.419 MPa</td>
<td>0.717 mm</td>
<td>4.195 MPa</td>
</tr>
</tbody>
</table>

Figure 2.11 shows how the taper angle affects both the maximum displacement and the mass of the two main links. The results indicate that the amount of reduction in the displacement is considerable when compared to the increase in mass. It shows that as the taper angle is increased, the percentage decrease in the displacement relative to the
percentage increase in the mass is reduced. Therefore, the stiffening and strengthening advantages of the taper angle is better realized at smaller taper angles. Lastly, the rate of reduction in the displacement in comparison to the increase in the mass is very similar for both the I-beam supported and X-supported rectangular tubes. As a result, neither cross-section can be chosen over the other without further analysis being performed on them that includes a torsional load due to the end-effector.

![Graph showing End-Effector Displacement Versus Mass for Various Taper Angles](image)

**Figure 2.11: End-Effector Displacement Versus Mass for Various Taper Angles**

### 2.3 Optimization of the Cross-Section

The accuracy of the robotic arm that is to be created is of great concern. As previously mentioned, current industrial-sized robotic arms are made of steel or aluminum to ensure high stiffness with the end result of providing good accuracy and repeatability. Therefore, in creating a robotic arm constructed of polymer composites, it is essential
to ensure that the stiffness-to-weight ratio is as high as possible. Taking this into consideration, along with the fact that the length of the robotic arm has already been specified and a manipulator layout has already been chosen that provides a large workspace, the stiffness of the robotic arm will be minimized with the use of an optimization procedure. The best way to achieve this is by altering the cross-section dimensions.

2.3.1 Determination of the Objective Function

In order to ensure that the optimization methodology arrives at a suitable solution for the dimensions of both the I-beam and X-supported rectangular cross-sections, the robotic arm needs to be analyzed under the worst possible loading scenario. Therefore, the objective function and constraints for this optimization problem were developed for the robotic arm with its two main links parallel, extending outwards fully, and the wrist configuration extended out fully to either side such that it was perpendicular to the main links as demonstrated in Figure 2.12. Additionally, the payload, \( W \), that is to be supported by the end-effector is applied at the end of the wrist such that in one direction it is the length of the two main links, \( L \), away from the support, and in the other it is the length of the wrist configuration, \( L_{\text{wrist}} \), away from the support. Consequently, the inclusion of both torsion and bending must be accounted for in developing the optimization methodology.

![Figure 2.12: Configuration of Robotic Arm to be Analyzed for Cross-Section Optimization](image)
In order to determine the objective function to minimize the deflection, the equation for the deflection experienced by the end-effector must be developed. With the robotic arm being analyzed in the position demonstrated in Figure 2.12, deflection occurs due to both bending and torsion. In fact, there is deflection associated with the bending of the two main links, bending of the wrist configuration, and twisting of the two main links. As a result, the objective function has to be defined in terms of these three components of deflection. Consequently, the concept of superposition was utilized to create the objective function.

In order to successfully analyze all the forces acting on the arm, it is first necessary to provide a full overview of the robotic arm. Since the arm being analyzed has two main links, there will be a motor controlling the second main link that is located between the two main links. Consequently, the weight of this motor will contribute to the deflection of the robotic arm. Furthermore, it is assumed that the two main links are of equal length and thus this force is acting in the middle of the total length of the two main links. Additionally, the weight of the links themselves also contributes to the deflection. Lastly, the payload that is applied at the end-effector (at the end of the two main links) also contributes to the amount of deflection as is illustrated in Figure 2.13.

![Figure 2.13: Side View of Free Body Diagram of the Analyzed Robotic Arm](image)

Figure 2.13: Side View of Free Body Diagram of the Analyzed Robotic Arm
The deflection of the robotic arm due solely to the bending of the two main links is first analyzed. From observing the free body diagram of the robotic arm as shown in Figure 2.13, the situation of bending of the two main links is demonstrated in Figure 2.14. Figure 2.14 illustrates that the distributive load of the weight of the links, and concentrated load of the weight of the motor and the payload all contribute to the deflection of the arm. As a result, the deflection due to the bending of the two main links, $\Delta Y_{\text{main links}}$, is a result of the sum of the three forces acting on the two main links and is denoted by [41]:

$$\Delta Y_{\text{main links}} = \frac{gw_{\text{link}}L^4}{8EI} + \frac{5gm_{\text{motor}}L^3}{48EI} + \frac{gWL^3}{3EI} \quad (2.2)$$

where $E$ is the elastic modulus, $I$ is the moment of inertia, $L$ is the total length of the two links, $g$ is the acceleration due to gravity, $W$ is the weight of the payload, $m_{\text{motor}}$ is the weight of the motor and $w_{\text{link}}$ is the weight of the link as denoted by:

$$w_{\text{link}} = \frac{\rho A_c L}{L} \quad (2.3)$$

Where $A_c$ is the cross-sectional area and $\rho$ is the density of the composite material.

The deflection of the robotic arm due to the bending of the wrist configuration, $\Delta Y_{\text{wrist}}$, is shown in Figure 2.15. It was assumed that the length of the wrist configuration was only one tenth of the length of the two main links. Consequently, the
weight of the links of the wrist configuration was extremely small relative to the weight of the payload and thus it was assumed to be negligible. Therefore, the payload is the only force acting on the wrist configuration and thus the only force contributing to the bending of the wrist. Thus, the deflection due to the bending of the wrist is:

\[
\Delta Y_{\text{wrist}} = \frac{gWL_{\text{wrist}}^3}{3EI}
\]  

(2.4)

Where \( L_{\text{wrist}} \) is the length of the wrist configuration.

![Figure 2.15: Free Body Diagram for Bending of the Wrist Configuration](image)

The deflection of the robotic arm due to the twisting of the main links, \( \Delta Y_{\text{twist}} \), is a result of the torque that is generated due to the payload being applied at a perpendicular distance to the main links. This is due to the fact that the wrist configuration is oriented such that it is stretched outwards to the side, perpendicular to the main links. The resulting torque acting on the main links is represented in Figure 2.16. The applied torque results in the main links twisting by an angle \( \phi \) as demonstrated in Figure 2.17. Using basic trigonometry, the assumption of a small angle \( \phi \), and the diagram shown in Figure 2.18, the equation for the displacement due to the twisting of the two main links is determined as:

\[
\Delta Y_{\text{twist}} = L_{\text{wrist}} \sin(\phi) \cos(\phi)
\]  

(2.5)

where \( \phi \) is the angle of twist.
Figure 2.16: Free Body Diagram for Torque Applied on the Main Links

Figure 2.17: Resulting Deflection of the Wrist Configuration due to Twisting of the Main Links

Figure 2.18: Geometric Representation of Deflection due to Twisting of the Main Links
The total deflection of the robotic arm is thus equal to the sum of the deflections that result due to bending of the main links and the wrist configuration, as well as twisting of the main links. Therefore the objective function is the total deflection ($\zeta$) of the robotic arm:

$$\zeta = \Delta Y_{\text{main links}} + \Delta Y_{\text{twist}} + \Delta Y_{\text{wrist}}$$  \hspace{1cm} (2.6)

### 2.3.2 Design of the Proof-of-Concept Prototype

The optimization methodology is carried out on a proof-of-concept prototype of the industrial-sized robotic arm that is to be designed. As of this point, there are too many variables concerning the design of the industrial-sized robotic arm (mostly surrounding the motors), and thus the optimization methodology was initially developed for the proof-of-concept prototype.

The prototype is to be powered using PowerCube rotary motors by Schunk [42]. A PR110 motor moves the first main link and the PR090 motor moves the second main link. Using a mass of 3.2 [kg] for the PR090 and a maximum output torque of 206 [Nm] and 425 [Nm] for the PR090 and the PR110, respectively, the ideal total length of the two main links was determined. It was assumed that the connection device for attaching the PR090 motor to the two main links has a mass of 1.5 [kg]. Using an efficiency of 90\% for the maximum output torque of the motors and the free body diagram of the main links with the torque of the motors and the force of the masses being applied on the links as illustrated in Figure 2.19, the maximum payload was determined.

It was determined that the maximum output torque of the PR110 was the limiting factor over that of the PR090 when the two main links are of equal length and thus the maximum payload, $W$, is determined using the maximum output torque of the PR110 motor and is calculated as:
\[ W = \frac{e f f M_{\text{max}}}{Lg} - \frac{L g m_{\text{motor}}}{2} - \frac{L^2 g \rho A_c}{2} \]  \hspace{1cm} (2.7)

where \( M_{\text{max}} \) is the maximum output torque of the PR110 motor and \( e f f \) is the efficiency at which the motor operates. In determining \( W \), the X-supported rectangular tube cross-section was used since for an equal perimeter it is heavier than the I-beam supported rectangular tube. Additionally, the dimensions that were used in performing the FEA study in Section 2.2 were used.

The maximum payload was calculated at various lengths for the two main links using dimensions for the cross-section that were proportional to the length of the two main links. The maximum payload was compared to the payload of the robotic arm that was proportional to the ratio of the size of the prototype to the industrial-sized arm and it was determined that the ideal length for the two main links was 1.75 [m], 35% the size of the industrial sized robotic arm, as shown in Table 2.2.

For a prototype that is 35% the size of the industrial sized robotic arm, the total length of the main links and the payload are 1.75 [m] and 17.5 [kg], respectively. Additionally, the length of the wrist was 0.175 [m] using the assumption that the wrist configuration is one tenth the length of the main links. Therefore, with the knowledge that the mass of the motor is equal to the mass of the PR090 motor and
the connection device, 4.7 [kg], it is clear that the objective function for deflection, Equation (2.6), is in terms of the area ($A_c$), moment of inertia ($I$), and angle of twist ($\phi$). Since the I-beam and X-supported rectangular tubes have different cross-sectional geometries, it is necessary to determine the area and moment of inertia for both cross-sections.

Table 2.2: Comparison of Length of Two Main Links to Prototype Payload

<table>
<thead>
<tr>
<th>Length of Two Main Links</th>
<th>Max Payload at 90% Max Output Torque</th>
<th>Max Payload as % of Intended Payload</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.5</td>
<td>21.67</td>
<td>1.44</td>
</tr>
<tr>
<td>1.55</td>
<td>20.77</td>
<td>1.34</td>
</tr>
<tr>
<td>1.6</td>
<td>19.92</td>
<td>1.24</td>
</tr>
<tr>
<td>1.65</td>
<td>19.11</td>
<td>1.16</td>
</tr>
<tr>
<td>1.7</td>
<td>18.35</td>
<td>1.08</td>
</tr>
<tr>
<td>1.75</td>
<td>17.63</td>
<td>1.01</td>
</tr>
<tr>
<td>1.8</td>
<td>16.95</td>
<td>0.94</td>
</tr>
<tr>
<td>1.85</td>
<td>16.30</td>
<td>0.88</td>
</tr>
<tr>
<td>1.9</td>
<td>15.68</td>
<td>0.83</td>
</tr>
<tr>
<td>1.95</td>
<td>15.09</td>
<td>0.77</td>
</tr>
<tr>
<td>2.0</td>
<td>14.52</td>
<td>0.73</td>
</tr>
</tbody>
</table>

2.3.3 Formulation of the Equations for the Cross-Sectional Area

The equations for the cross-sectional area for both geometries are determined using the concept of combination of composite parts. The I-beam supported rectangular tube can be considered as a large rectangle minus two smaller rectangles of equal dimension as shown in Figure 2.20. Using this approach the equation for the area for the I-beam supported rectangular tube is determined as:

$$A_c = 2wt + 3ht - 6t^2$$  \hfill (2.8)

where $h$, $w$, and $t$ represent the height, width, and wall thickness of the link.
Figure 2.20: Cross-Sectional Area of the I-Beam Supported Rectangular Tube

The same procedure was utilized for determining the cross-sectional area for the X-supported rectangular tube. Since the thickness of all the walls and interior support features is defined to be the same thickness \( t \), the area of the different components comprising the X-supported rectangular tube involve some calculations. Figure 2.21 illustrates the cross-section and the distances that need to be calculated before the dimensions of the basic components can be determined.

Figure 2.22 shows a close up view of the areas of concern. Using basic trigonometry, the equations for the distances of \( x_h \), \( y_h \), \( x_v \), and \( y_v \) were determined as:

\[
x_h = \frac{t}{2 \cos(\tan^{-1}\left(\frac{w}{h}\right))}
\]

\[
y_h = \frac{t \cos(\tan^{-1}\left(\frac{w}{h}\right))}{\cos(\tan^{-1}\left(\frac{h}{w}\right) - \tan^{-1}\left(\frac{w}{h}\right))}
\]

\[
x_v = \frac{t \sin(\tan^{-1}\left(\frac{w}{h}\right))}{\cos(\tan^{-1}\left(\frac{h}{w}\right) - \tan^{-1}\left(\frac{w}{h}\right))}
\]

\[
y_v = \frac{t}{2 \cos(\tan^{-1}\left(\frac{h}{w}\right))}
\]
Figure 2.21: Cross-Section of the X-Supported Rectangular Tube

Figure 2.22: Middle and Corner of Cross-Section for the X-Supported Rectangular Tube
The X-supported rectangular tube can be considered as a rectangle minus two horizontal triangles minus two vertical triangles as demonstrated in Figure 2.23. Therefore the cross-sectional area for the X-supported rectangular tube is the area of the rectangle minus two times the area of the horizontal triangle minus two times the area of the vertical triangle. Thus the equation for the cross-sectional area for the X-supported rectangular tube is:

\[ A_c = wh - (w - 2t - 2x_h)(\frac{h}{2} - t - y_h) - (h - 2t - 2y_v)(\frac{w}{2} - t - x_v) \] (2.13)

2.3.4 Formulation of the Equations for Moment of Inertia

The equations for the moment of inertia for the two different cross-sectional geometries can also be determined with the use of the combination of composite parts. By reasons of symmetry, the neutral axis for the bending of the main links about the X-axis is located at the midheight of the links cross-section for both cross-sectional geometries as demonstrated in Figure 2.24. By using the equations for the area of the different base components of the cross-section that were determined in the previous section, the moment of inertia for those base components was determined.
For the I-beam supported rectangular link, the moment of inertia for the cross-section is determined by subtracting two times the value of the moment of inertia of the small rectangle from the moment of inertia of the large rectangle as is shown in Figure 2.25. As a result of the symmetry of the base components of the I-beam supported rectangular cross-section, the neutral axis of the large and small rectangles is coincident with the neutral axis of the entire cross-section. Consequently, the moment of inertia for this cross-sectional geometry is:

\[
I = \frac{1}{12} wh^3 - \frac{1}{6} \left(\frac{w - 3t}{2}\right)(h - 2t)^3
\]

(2.14)
The same procedure that was utilized to determine the moment of inertia for the I-beam supported rectangular link was used to determine the moment of inertia for the X-supported rectangular link. The equations for the dimensions of the different base components that make up the X-supported rectangular link were used to determine the equation for its moment of inertia. As demonstrated in Figure 2.26, the moment of inertia for this cross-sectional geometry is equal to the moment of inertia of the rectangle minus two times the moment of inertia of the horizontal triangle minus two times the moment of inertia of the vertical triangle about the neutral axis of the cross-section.

The neutral axis of the horizontal triangle is not coincident with the neutral axis of the structure and thus the parallel axis theorem must be employed. For the vertical triangle, since it is perpendicular to the horizontal triangle, its dimension for height is in the same direction as the width of the structure. As a result, its y-axis is coincident with the neutral axis of the structure and thus the moment of inertia for the vertical triangle is computed about its y-axis as [41,43]:

\[
I_x = \frac{1}{12} b_{\text{rect}} h_{\text{rect}}^3 = \frac{1}{12} w h^3
\]

\[
-2 \left[ \frac{1}{36} \left[ w - \left( 2 + \frac{1}{\cos(\tan^{-1}(w/h))} \right) t \right] \right]^{3/2}
\]

\[
-2 \left[ w - \left( 2 + \frac{1}{\cos(\tan^{-1}(w/h))} \right) t \right] \left[ \frac{h}{2} - \left( 1 + \frac{\cos(\tan^{-1}(w/h))}{\cos(\tan^{-1}(h/w) - \tan^{-1}(w/h))} \right) t \right]^{3/2}
\]

\[
-2 \left[ \frac{1}{48} \left[ h - \left( 2 + \frac{1}{\cos(\tan^{-1}(h/w))} \right) t \right]^{3/2} \right]
\]

\[
-2 \left[ h - \left( 2 + \frac{1}{\cos(\tan^{-1}(h/w))} \right) t \right] \left[ \frac{w}{2} - \left( 1 + \frac{\sin(\tan^{-1}(w/h))}{\cos(\tan^{-1}(h/w) - \tan^{-1}(w/h))} \right) t \right]^{3/2}
\]

(2.15)
2.3.5 Formulation of the Equations for Angle of Twist

The final unknown in the objective function is the angle $\phi$, which denotes the angle of twist of the main links. For a thin-walled tube with a closed cross section and constant thickness the angle of twist is defined as [41]:

$$\phi = \frac{TL \oint ds}{4GA_m t}$$  \hspace{1cm} (2.16)

where $T$ is the internal torque acting on the links at the wrist location, $\oint ds$ is the line integral around the entire boundary of the links’ cross-sectional area, $G$ is the shear modulus of the composite material, and $A_m$ is the mean area enclosed within the boundary of the centerline of the links’ thickness.

However, this equation is for closed cross-sections that do not include any interior support structure. Therefore, in order to utilize this equation it is necessary to alter the I-beam and X-supported rectangular links into an equivalent hollow rectangular link. As is demonstrated in Figure 2.27, the equivalent hollow rectangle has the same width and height as that of the I-beam and X-supported rectangular links. Therefore the thickness of the equivalent rectangle is increased to account for improvement in the torsional rigidity that is provided by the I-beam or X-support structures. This equivalent thickness is calculated by setting the cross-sectional area of the equivalent
rectangle to that of either the I-beam or X-supported rectangular link (whichever is being optimized) and rearranging and solving for the equivalent thickness, $t_{equiv}$. The equation for the resulting equivalent thickness is:

$$
    t_{equiv} = \frac{2(w + h) - \sqrt{(2(w + h))^2 - 16A_c}}{8}
$$  \hspace{1cm} (2.17)

Figure 2.27: Equivalent Hollow Rectangle for the Ideal Cross-Sectional Geometries

The mean area, $A_m$, for the equivalent rectangle is shown in Figure 2.28. Observation of the figure indicates that the mean area is defined as:

$$
    A_m = (w - t_{equiv})(h - t_{equiv})
$$  \hspace{1cm} (2.18)

For a rectangular cross section the line integral, $\int ds$, is simply the perimeter of the centerline boundary of the thickness:
\[ \int ds = 2(w - t_{equiv}) + 2(h - t_{equiv}) \]  

(2.19)

Figure 2.28: Mean Area Enclosed within Boundary of Centerline for Equivalent Hollow Rectangle

\section*{2.3.6 Determination of Constraints}

With the use of the equations derived in the previous section, the equation for the total deflection in the robotic arm ($\zeta$) as illustrated in Equation (2.6) is in terms of the cross-sectional dimensions: height ($h$), width ($w$), and thickness ($t$). Considering the equation for the total amount of deflection is the objective function, the variables for the optimization problem are $h$, $w$, and $t$.

The constraints for this optimization problem still need to be defined. By analyzing the geometry, a few constraints can be created. Considering that the thickness of the X-support structure is the same as that of the outer rectangle, it is necessary that the width is greater than or equal to four times the thickness. If this is not the case, then it allows the possibility of the different components overlapping. As a result, one would end up with either a solid part or one in which it is far too difficult to differentiate the X-support from the outer rectangular tubing.

For the case of the I-beam supported rectangular structure, as long as the width is
more than three times that of the thickness then the part will not be solid. However, for convenience sake the same constraint for the X-supported rectangular link is utilized.

From the results shown in Section 2.2.2, it was determined that a rectangular shape in which the height is greater than the width is favoured over a square cross section. Consequently, to minimize the total deflection of the robotic arm the height should be greater than or equal to the width. The equations for these two linear inequality constraints are:

\[
\begin{align*}
    w & \geq 4t \quad \Rightarrow \quad 4t - w \leq 0 \\
    h & \geq w \quad \Rightarrow \quad w - h \leq 0
\end{align*}
\] (2.20)

\[
\begin{align*}
    h & \geq w \quad \Rightarrow \quad w - h \leq 0
\end{align*}
\] (2.21)

Equations (2.20) and (2.21) can be put in the form \(Ax - b:\)

\[
\begin{bmatrix}
    -1 & 1 & 0 \\
    0 & -1 & 4
\end{bmatrix}
\begin{bmatrix}
    h \\
    w \\
    t
\end{bmatrix}
\leq
\begin{bmatrix}
    0 \\
    0
\end{bmatrix}
\] (2.22)

In addition to the aforementioned linear constraints, the maximum output torque of the motors can be used to create a constraint. As previously mentioned, the maximum output torque of the PR110 motor which is connected to the first main link is the limiting value. As a result, the maximum output torque of the PR110 is only needed for developing the constraint that ensures that the torque that is generated by the motor is sufficient to keep the robotic arm stable.

The main links were analyzed under the situation in which the maximum output torque provided by the PR110 motor was being applied. Additionally, since this
situation involves analysis of forces only resulting in bending of the main links it is analyzed with the payload acting directly at the end of the main links, not at an offset equal to the length of the wrist. Furthermore, efficiency was included for the maximum output torque of the motors to ensure that even with the motor not operating at full power, the robotic arm will remain stable. This is the same loading scenario that is illustrated in Figure 2.19. Using this free body diagram of the situation and the consideration that the efficiency multiplied by the maximum output torque of the PR110 motor must be greater than or equal to the sum of the torques that result due to the forces acting on the links, the following non-linear constraint is developed:

\[ A_c + \frac{2}{L\rho} \left( \frac{m_{\text{motor}}}{2} + W \right) - \frac{2 \text{eff} M_{\text{max}}}{L^2 g \rho} \leq 0 \]  

(2.23)

In Equation (2.23), all the terms but the cross-sectional area are known. Since the cross-sectional area of both the I-beam and X-supported rectangular links are in terms of the height, width, and thickness, these are the variables for the constraint. By looking at the equations for cross-sectional area for both the I-beam and X-supported rectangular geometries (Equations (2.8) and (2.13), respectively), it is easy to see that at least one of the three variables is raised to a power greater than one. As a result, the constraint defined by Equation (2.23) is non-linear.

By observing the objective function, Equation (2.6), for this optimization problem it is easy to see that it is a non-linear multivariable function. Additionally, there are both linear and non-linear multivariable constraints (Equations (2.22) and (2.23)). The MATLAB *fmincon* solver from the Optimization Toolbox [44] was used to find the optimal solution for \( h, w, \) and \( t \).
2.3.7 Determination of Bounds for the Design Variables

The *fmincon* solver can also utilize upper and lower bounds on the variables to achieve the best results possible. Considering that the PR090 motor measures 90 [mm] by 90 [mm] on each of its faces, the upper bound for the height and width of the link was set at 140 [mm] such that the link is not considerably larger than the motor. Similarly, the lower bound used was 45 [mm], half the dimension of the PR090 motor. As for the thickness of the link, the lower bound was chosen as 2 [mm] because prior studies indicated that for best results no fewer than 10 layers of composite should be used. Additionally, the thinnest laminates that are available and that are not difficult to work with are 0.18 [mm] thick [45]. As for the upper bound of the thickness, it should not be more than a quarter the lower bound of the width in order to ensure that the linear constraint shown in Equation (2.20) is satisfied. Therefore, an upper bound of 10 [mm] was selected for the thickness.

2.3.8 Material Specifications

The optimization methodology was performed assuming that the links were constructed of high modulus carbon fiber-epoxy matrix with a 60% fiber volume. It was modeled as an isotropic material with the same properties as those used in the preliminary FEA of Section 2.2. Thus a density (*ρ*) of 1,700 [kg/m³], elastic modulus (*E*) of 220 [GPa], and Poisson’s ratio (*ν*) of 0.25 were used.

Due to the highly longitudinal loading of the link, the shear modulus (*G*) was determined using the shear modulus equation for isotropic materials [5]:

\[ G = \frac{E}{2 \times (1 + \nu)} \]  

(2.24)

Using the aforementioned values, the shear modulus for the polymer composite material was calculated to be 88 [GPa].
2.3.9 Solutions Found from Optimization of Cross-Section

2.3.9.1 I-beam Supported Rectangular Link

Using the constants that have been declared and the equations derived throughout this section, the optimization solution was achieved (please see Appendix A to view the complete MATLAB coding). For the I-beam supported rectangular link, the optimization methodology was used at a variety of different starting points for the height, width, and thickness ranging anywhere from above the upper bounds to below the lower bounds. In all cases, an identical solution was determined in a different number of iterations. The solution point that was found was 140 [mm] for both the height and width and 2.3829 [mm] for the thickness. This equates to a minimal end-effector displacement of 0.37146 [mm].

The values from the solution point for the height, width, and thickness were substituted into the linear constraints (Equation (2.20) and (2.21)) as well as the non-linear constraint (Equation (2.23)). All three equations were satisfied. This indicated that the solution point that was returned was indeed a valid solution.

The non-linear constraint returned a value that was zero to at least 11 decimal places. Inspection of Equation (2.23) reveals that a result for this equation that is essentially zero is a result of the cross-sectional area being set as the maximum allowable amount. This seems quite natural since increasing the size of the link and amount of material used to produce it leads to an increase in the stiffness and consequently a decrease in the deflection.

The solution point for both the height and width are at the upper bounds, whereas the thickness is just slightly greater than the lower bound. This indicates that the ideal solution for minimal displacement is to increase both the height and width while decreasing the thickness. This makes sense since objects with thin sections like I-beams and other popular structural geometries have a very high moment of inertia. The thickness is greater than the lower bound because the upper bounds of both the
height and width make it impossible to have a cross-section that is the maximum allowable amount with the minimum thickness.

### 2.3.9.2 X-Supported Rectangular Link

As was the case for the I-beam supported rectangular link, the optimization methodology for the X-supported rectangular link was carried out for a variety of starting points that ranged from anywhere above the upper bounds to below the lower bounds. An identical solution of 140 [mm] for the height, 105.24 [mm] for the width, and 2.0 [mm] for the thickness was obtained. This equates to a minimal deflection for the end-effector of 0.44736 [mm]. The solution point found satisfied the linear and non-linear constraints indicating that it is a valid solution.

The solution point for the X-supported rectangular link also has the maximum permissible cross-sectional area. The solution for the height was found to be equal to the upper bound, whereas the width was found to be smaller than the height. Additionally, the thickness at the solution point was equal to the lower bound value. Therefore, this indicates that the thickness of the X-supported rectangular link is minimized and then the height is increased relative to the width. This agrees with the results found from the preliminary FEA in Section 2.2 in that it also determined that a rectangular cross section was found to have less deflection than a square cross section. Considering the majority of the loading on the robotic arm is longitudinal for this application, it is reasonable that a link with a higher ratio for the height to the width is preferred.

### 2.3.9.3 Study Without Upper Bounds

For both cross-sections, the solution point was restricted to the upper bounds. Therefore, to gain a better understanding of the optimal dimensions for the cross section the optimization methodology was carried out without these limiting upper bounds. It
was tested at a variety of different starting points that returned the same solution for the most part. Alternative solutions were ignored either because they did not satisfy the linear and non-linear constraints or the corresponding deflection was greater than that found for the true optimal solution.

The minimal deflection for the I-beam supported rectangular link was found to be 0.20192 [mm] for a height of 233.50 [mm], width of 64.24 [mm], and thickness of 2.0 [mm]. The thickness was minimized to the lower bound of 2.0 [mm] and the height was increased relative to the width such that it was around 3.63 times as large.

The minimal deflection for the X-supported rectangular link was determined to be 0.34535 [mm]. The solution point has a height of 184.79 [mm], width of 49.93 [mm], and a thickness of 2.0 [mm]. Similar to the I-beam supported rectangular link, the thickness was minimized and the height was increased to be 3.70 times the width of the link.

2.3.9.4 Recommendations from Optimization Results

Upon carrying out the optimization methodology described, it was determined that with a combination of both torsion and bending, the I-beam supported rectangular link results in less deflection compared to the X-supported rectangular link. For the proposed application, the amount of bending is considerably larger than the amount of torsion and thus it seems sensible that the I-beam interior support structure results in a lower amount of deflection.

The ratio of the height to the width was greater for the X-supported rectangular link since it has greater torsional rigidity as a result of its structure. Consequently, its height can be increased relative to the width to further increase its longitudinal stiffness. As for the I-beam supported rectangular link, the presence of the I-beam provides less torsional rigidity and in order to provide a significant amount of torsional rigidity, the height-to-width ratio needs to be less than that of the X-supported
rectangular link. However, an X-support requires more material than an I-beam. Therefore, the perimeter of the rectangle must be smaller for the X-supported rectangular link in order that the constraints are met. This results in a lower moment of inertia and thus the bending stiffness is sacrificed.

For both structures the solutions from the optimization methodology indicated that in order to improve the stiffness and thus reduce the amount of deflection experienced in the robotic arm the following modifications should be made to the cross-section in the sequence described. First off, the cross-sectional area should be maximized. Secondly, the thickness should be minimized. Lastly, the height should be increased as the width is decreased in order to arrive at a link that provides considerably greater bending stiffness than torsional rigidity.

2.4 Summary

A serial main configuration with a $R \perp R \parallel R$ main-arm layout was chosen for the 6-DOF robotic arm. The cross-section for the links was determined to be rectangular with an I-beam support in the middle. FEA revealed that it is optimal to include a negative taper angle of less than one degree. A 35\% size proof-of-concept prototype was determined to have minimal deflection for height, width, and wall thickness of 233.50 [mm], 64.24 [mm], and 2.0 [mm], respectively.
Chapter 3

Prototype Link Construction

In this chapter, the construction of the prototype link is described in detail. A novel multi-segment link design and method of production was created. A suitable layering pattern was chosen based on the requirements of the application for the robotic arm. The creation of joints for mounting the links to the PR110 and PR090 motor is also discussed. In the end, a 2-DOF main-arm proof-of-concept prototype was created 1.

3.1 Geometry

In order to test the capabilities and performance of a polymer composite robotic arm, a prototype was constructed. As mentioned in Section 2.1.2, the two main links provide the majority of the reach for the desired 6-DOF robot. Therefore, for testing purposes a 2-DOF robot main-arm representing these links was constructed. From the results of the FEA and optimization outlined in Chapter 2, the links for the prototype were constructed to have an I-beam supported rectangular cross-section. The two links were both designed to be 87.5 [cm] in length. As determined from the optimization procedure, the height, width, and thickness of the prototype links were designed to be 233.5 [mm], 64.24 [mm], and 2.0 [mm], respectively. The links were

1 Please see [46] for paper on this topic
constructed with a negative taper angle of $0.5^\circ$. However, for ease of production, the prototype links were designed to only be tapered in one direction. The taper angle was relative to the longitudinal axis such that the height of the link was tapered. Therefore, the height at the end of the first link and beginning of the second link was designed to be 233.5 [mm]. Consequently, the height at the beginning of the first link and at the end of the second link were designed to be 248.75 [mm] and 218.25 [mm], respectively.

### 3.2 Modular Construction Process

The construction of the prototype links was done using a modular process. This novel process involves creating a series of identical hollow modular base units using the same mold. The modular base units are then assembled together in the proper orientation to create the desired interior support structure of the component. The exterior profile is then constructed directly around the modular base units. Therefore, the assembly of the base units is used as the mold for the construction of the exterior profile as is illustrated in Figure 3.1.

This technique is favourable because it requires only one simple mold for producing the hollow base units. It is considerably less expensive than a complicated mold that would be required to produce the link in one step. Additionally, this technique produces a stronger bond between the base units and the exterior profile. As the epoxy hardens and creates the solid exterior profile, it cures directly to the base units.

The I-beam supported rectangular link consists of two rectangular base units. Therefore, for the production of the prototype links one rectangular link half was created on a rectangular mold. The mold was removed and the second rectangular half was constructed. The two halves were then placed together and the exterior profile was constructed around them.
3.3 Mold Construction

3.3.1 Phase 1: Wooden Mold

In order to produce the rectangular halves for the I-beam supported rectangular links, two solid wooden molds, one for each link, were constructed for reasons of economics and ease of production. The molds were constructed in order to produce two prototype links with the dimensions as stated above. The wooden molds were constructed out of medium density fiberboard (MDF). MDF was chosen because it is less expensive than natural wood. It was selected over particle board and plywood because it is less likely to chip apart when being cut and shaped.

The MDF was cut to size for producing both prototype links. The edges of the wood were routed to aid in the removal of the mold. Additionally, the rounded edges produce a link that is preferred over sharp corners that act as stress concentrators. Numerous layers of Duratec primer and sealer were applied to the molds. After each layer had been applied and dried, the molds were sanded. This ensured that there were no bumps that would make removal more difficult. The wooden mold for the construction of one of the links is pictured in Figure 3.2.
Despite all the provisions taken to aid in the removal of the mold, the cured composite rectangular part could not be separated from the wooden mold. Failure of removal for a second time led to a revision of the mold.

### 3.3.2 Phase 2: Four Piece Fully-Detachable Aluminum Mold

In order to save both time and money, only one mold was created for the production of both prototype links. To this end, the prototype links were designed to be the exact same size. Also, the four piece mold was not designed to the same dimensions as the wooden mold. The rectangular base unit that was created using the wooden mold had a thickness of approximately 1.7 [mm]. This thickness represents half the total thickness and thus a complete link would have a thickness of around 3.4 [mm]. This is considerably larger than the prescribed 2.0 [mm]. Therefore, if it were to be made with the prescribed height and width then the composite prototype links would be too heavy. Furthermore, the relatively low torsional rigidity of the rectangular base unit seemed to contribute to the difficulty in removing the link. As a result, the ratio of the height to the width was decreased to improve its torsional strength. Lastly, in order to ease assembly, the width of the link was designed to be slightly smaller than prescribed to ensure that it fit between the mounting holes of the PR090 motor. As a result, the dimensions of the mold were such that it created a link that has a mid-height and width of 120 [mm] and 60 [mm], respectively.
Since the removal of a solid mold with only a -0.5° taper was not successful, a fully-detachable mold was created. Four flat aluminum plates that are combined to produce the rectangular mold were used. The plates were kept together using screws and nuts and bolts that can be taken apart to allow for disassembly.

This mold relied on compliance to enable the removal of the parts of the mold. Therefore, the mold was constructed out of aluminum because of its high flexibility and relatively low cost. Since it is very flexible, a wooden skeleton was placed inside the combination of the aluminum plates as illustrated in Figure 3.3. The skeleton helped to keep the shape of the mold and prevent the aluminum plates from collapsing in on themselves and thus altering the shape of the link being constructed.

![Figure 3.3: Four Piece Fully-Detachable Mold](image)

Aluminum is non-porous and therefore it does not allow the epoxy to permeate the mold which makes removal more difficult. Therefore, it was not necessary to apply primer and sealer to the mold.

Upon creation of the composite rectangular base unit, the four piece mold was successfully detached from the cured part. However, the resulting composite part was of poor quality. The part had many variations and warpages in its rectangular profile. The top and bottom aluminum plates did not hold their shape very well as the vacuum force pulled them towards it, creating a link with severe bowing on the top and bottom surfaces. Therefore, the mold had to be modified so that it was flexible
enough to enable removal of the part but rigid enough to keep its shape during the composite link production.

### 3.3.3 Phase 3: Two Piece Fully-Detachable Aluminum Mold

In order to eliminate the possibility of the top and bottom pieces flexing as the part was cured, they were made rigid. Therefore, in order to still offer full disassembly of the mold, two identical L-shaped halves were used for the mold in place of the four aluminum plates as illustrated in Figure 3.4. The aluminum plate was bent such that the side of the mold and the top of the mold were one solid piece. Similarly, the other side wall and the bottom of the mold were one continuous piece. The same wooden skeleton that was used for the four piece mold was used to ensure that the mold held its shape. The end-result was a composite part with a relatively smooth profile that was capable of being successfully removed from the mold.

![Figure 3.4: Two L-Shaped Piece Fully-Detachable Mold](image)

### 3.4 Prototype Link Construction

#### 3.4.1 Technique

Based on the fact that a modular construction process as described in Section 3.2 was to be utilized, the links were constructed as a series of hollow members. As a result,
the filament winding process is well-suited for the production of the links. Therefore it is envisioned that the industrial-grade robotic arm will be constructed using filament winding. However, the 2-DOF robot main-arm proof-of-concept prototype was developed using hand lay-up for reasons of economics.

The production of the polymer composite link was achieved using vacuum bag molding. This was used in conjunction with hand lay-up because it helps to reduce the amount of epoxy over hand lay-up alone. The vacuum draws away the excess epoxy from the fibers. This results in a final composite product that has a greater percentage of fibers. This is ideal because they provide the reinforcing properties to the composite whereas the epoxy only holds the fibers in place.

Unlike the polymer composite robots discussed in Section 1.5.1, the prototype links were not created with the use of an autoclave. The prototype links were too large to safely fit in a conventional oven, and it was too expensive to purchase a suitable autoclave. For these reasons, the composite links were cured at room temperature.

### 3.4.2 Materials Used for Prototype

Considering the hand lay-up procedure was used for the production of the prototype links, a uni-directional twill manufactured by BGF Industries, Inc. was used. Additionally, the selected twill was uni-directional because it permits control over the mechanical properties of the composite by having complete control over the fiber orientation. Furthermore, the twill material used was T300 carbon fiber because, as discussed in Section 1.4.3.2 and outlined in Section 2.2.1, carbon fibers are preferred over aramid and glass fibers because they have the highest stiffness.

Epoxies were selected as the matrix material because they have better structural properties than other alternatives. PR2032, an epoxy manufactured by Aeropoxy, was used for the production because it cures at room temperature. In addition to this, PH3660, a methyl ethyl ketone peroxide (MEKP) catalyst also manufactured by Aeropoxy, was
used to improve the stiffness of the polymeric composite.

3.4.3 Ply Lay-up Pattern for Prototype

As mentioned in Section 2.3.9.4, the majority of loading for this application is bending. Therefore to provide improved strength for this loading, the majority of fibers were oriented parallel to the longitudinal axis ($0^\circ$) [25]. In addition to this, some layers should be oriented at an angle of $45^\circ$ to the longitudinal axis so as to promote strength against both torsional and shear loads [29]. Additionally, a small percentage of $90^\circ$ layers should be present to couple with the $0^\circ$ layers and improve the axial strength. Also, as indicated in [26], they help to improve the strength at the joint by reducing the hoop stress under vertical loads.

The percentage of $45^\circ$ layers is determined based on the $l/R$ ratio as outlined in [31]. Therefore for the I-beam supported rectangular prototype link, the Arm Ratio was determined as:

\[
\text{Arm Ratio} = \frac{L}{L/2 \cdot h_{avg}} \quad (3.1)
\]

where $h_{avg}$ is the average height of the link. Therefore, for a link length of 87.5 [cm] and average height of 233.5 [mm], the arm ratio is approximately 7.5. By observing Figure 1.8, the percentage of $45^\circ$ layers for maximum fundamental frequency is 25%. In order to meet the aforementioned specifications, 12 layers were used. Therefore, three of the layers were oriented at $45^\circ$ to the longitudinal axis. Also, the link was designed so that no more than 10% of $90^\circ$ layers were present. Therefore, only one $90^\circ$ layer was used. The remaining eight layers used were $0^\circ$ layers in order to provide sufficient bending stiffness. The order of fiber layers was chosen to be repetitive to provide an even distribution of the types of layers. The inner layer was $0^\circ$, followed by $45^\circ$, $0^\circ$, $0^\circ$, $45^\circ$, $0^\circ$, $0^\circ$, $90^\circ$, $0^\circ$, $0^\circ$, $45^\circ$, and finally concluding with another $0^\circ$ layer.
3.4.4 Manufacturing Procedure

As mentioned in Section 3.2, the prototype links were constructed by producing two rectangular base units first and then wrapping the exterior profile around them. Therefore, the rectangular base unit was created out of the first 6 plies of the unidirectional carbon fiber twill.

The mold was coated with six layers of mold release wax produced by T.R. Industries to ensure the release of the mold. Additionally, the edges where the L-shaped mold halves meet were sealed with tape to prevent the epoxy from entering this joint and bonding the halves together. The carbon fiber twill was then cut to the proper dimensions. Since the mold is tapered, the carbon fiber cannot be wrapped around the entire mold, and instead was cut into individual pieces for each side of the mold.

The epoxy and stiffener combination was then applied to the twill layers with the use of a paint brush. The layers were then placed onto the mold one at a time. A firm roller was then used to ‘roll out’ air bubbles and help ensure that the part produced did not have any voids in it.

Once all the layers had been placed on the mold, the vacuum bag apparatus was setup as illustrated in Figure 3.5 (this figure shows a cut away version with all components visible). Peel ply was wrapped around the carbon fiber twill followed by release film and breather fabric. The peel ply aided in the removal of the release film and the breather fabric that helped absorb the excess epoxy. The part was then surrounded by a vacuum bag that was fully sealed and connected to a vacuum pump. The vacuum pump was then turned on, sucking up the excess epoxy as it cured creating the hardened composite base unit. The vacuum bag apparatus was removed approximately six hours later to allow the part to fully cure.

Once cured, cotton filling was placed inside the mold and liquid nitrogen was poured inside. This decreased the temperature of the aluminum mold drastically, causing it to contract. The mold was then pushed out of the composite part, one half at a time.
The epoxy and release wax residue was cleaned off of the mold. The procedure was repeated, creating the second composite rectangular base unit. Epoxy was then applied to the two base units bonding them together. The final six plies were cut, soaked in epoxy, and placed on the assembled base units. The vacuum bag apparatus was setup again and the part was cured, creating a completed link. This entire procedure was then repeated, creating a second identical link.

### 3.5 Link Attachment Bracket

Once the composite prototype links were successfully produced, a bracket for attaching the link to the motors needed to be constructed. The brackets were made out of stainless steel because it has a very high modulus of elasticity. This helps to ensure that any deflection of the attachment device is negligible in comparison to that of the links. Additionally, carbon fiber is quite cathodic and can thus cause metals to corrode so a material that is not that corrosive was used [29]. Figure 3.6 shows the design of the bracket. It consists of six plates that enclose the exterior and interior support walls of the link. It also includes a series of spacers that are placed between the plates in the hollow portion of the link to keep the plates
butted up against the walls of the link. All the plates have four equally spaced holes in them. These holes were used to enable the connection of the bracket to the composite link with four bolts. The diameter of the holes in the plates and the link were made larger than the diameter of the fastening bolts so that stress concentrations were not created from the link resting on the bolts in order to prevent the link from being damaged. Consequently, the connection between the attachment device and the link is fully frictional.

![Figure 3.6: Bracket for Attaching Composite Link to the Joint Motor](image)

All of the plates of the bracket were clamped together and four holes were drilled. Then, one of the plates was placed on the link and the centers of the holes were marked off. Four holes were then drilled through the entire width of the composite link. Another set of four holes were drilled through the base plate at the corners. Two of the plates that attach to the exterior walls of the link were welded to the base plate to create a rigid connection. The spacers were glued to the plates to keep them in place while PR2032 epoxy was applied on the other side of the plates. The plates were then inserted into the hollow part of the link and lined up with the holes in the composite link. The fastening bolts were then inserted through the holes in the link and bracket and tightened to securely fasten the link to the attachment bracket. Finally, the motor was lined up with the base plate of the bracket and was fastened.
to it with bolts.

The final assembly of the motor with the composite link via the attachment bracket is illustrated in Figure 3.7. Another three brackets were created and fastened to the links and the motors to create a fully functional 2-DOF robot main-arm prototype. The motor that powers the first link of the 2-DOF prototype was securely mounted to the ground as is illustrated in Figure 3.8. The complete 2-DOF robot main-arm proof-of-concept prototype is shown in Figure 3.9.

Figure 3.7: Composite Link Connected to the Motor with Attachment Bracket

Figure 3.8: 2-DOF Main-Arm Proof-of-Concept Prototype Mounted to the Floor
Figure 3.9: 2-DOF Main-Arm Proof-of-Concept Prototype
3.6 Summary

Two identical carbon fiber-epoxy composite links with an I-beam supported rectangular cross-section were created. A novel manufacturing procedure which included producing simple hollow modular base units, assembling them, and wrapping around the entire assembly was utilized to simplify the production of the links. Steel attachment brackets were also manufactured. A 2-DOF main-arm proof-of-concept prototype was created by fastening the composite links to the motors with the attachment brackets.
Chapter 4

Prototype Link Testing

Once the composite links were created, testing was performed on the links. A variety of tests were performed to test both the static and dynamic capabilities of the carbon fiber-epoxy composite link. The procedure is described and the results are indicated and analyzed. Additionally, a virtual model of the prototype was created using Abaqus CAE. FEA was performed on the virtual model in order to test the results obtained for the physical prototype. The results and analysis of the tests are discussed in this chapter.

4.1 Physical Prototype Testing

4.1.1 Single Link Static Testing

Static testing was performed on a single link to determine the deflection of the link under a constant load. By using only one link, it permits a measurement of stiffness for the link itself that does not include any displacements that result due to the attachment brackets that connect the two composite links, nor due to the joint.
4.1.1.1 Weight Support Component

For the static testing, the composite link was studied as a cantilever beam. In doing so, a force was applied on the end of the link by adding weights to the link. Therefore, a component was created that could support the weight that was to be added on to the link. The weight that was placed on the end of the link were standard weighted discs used for weight-lifting. Therefore the weight support component included a hollow cylindrical tube that could be inserted through the inner hole of the discs. This cylindrical tube was connected to a base plate that has four holes that match the base plate of the attachment bracket. The design of the weight support component is illustrated below in Figure 4.1.

![Device for Supporting Weight on the Composite Link](image)

Figure 4.1: Device for Supporting Weight on the Composite Link

The center of the four mounting holes on the base plate of the attachment bracket were marked on a 9 [cm] by 9 [cm] stainless steel plate. The holes were then drilled. A hollow cylindrical stainless steel tube with a diameter of 25 [mm] and wall thickness of 3 [mm] was cut to a length of 12 [cm]. The tube was placed in the middle of the plate and was welded to it to create a rigid joint. The weight support component was then fastened to the attachment bracket with nuts and bolts. The dimensions of the component were chosen such that they matched up with the attachment bracket. The total weight of the two components combined was 3.85 [kg]. This weight made it possible to easily add weighted discs to achieve a total payload of 17.5 [kg].
4.1.1.2 Apparatus

As illustrated in Figure 4.2 the larger end of the composite link was rigidly attached to a cement wall with the use of the attachment bracket (see Section 3.5). This was achieved by drilling four holes in the wall and using cement anchor bolts to mount the bracket to the wall. By mounting the link to the wall rather than a motor, it eliminated any displacements that are a result of the motor, including any inadvertent rotation due to loading of the link. This enabled the measurement of the displacement that occurs due to the deflection experienced by the composite link only.

![Link Mounted to Wall](image)

Figure 4.2: Link Mounted to Wall

Figure 4.3 shows a schematic of the experimental setup used for the investigation of the static loading. As shown, at the far end of the composite link a laser was mounted such that it was 30 [cm] below the bottom of the link and perpendicular to the ground. The laser was shone near the end on the underside of the link and measured the vertical distance between the laser and the link.

The displacement was measured at four intervals. First, the link was tested with just the weight attachment device mounted on the end of the link. Measurements were then taken by incrementally adding the 4.55 [kg] (10 [lb]) weighted discs to the link. The measurements were taken 10 minutes after the weight had been added to allow the link to settle.
4.1.1.3 Results

The average values of the data collected for the vertical displacement from a series of five tests using the laser are shown in Table 4.1. The difference between each subsequent measurement is close to the same value; in fact the maximum percentage difference was less than 5%. This indicates a primarily linear displacement.

<table>
<thead>
<tr>
<th>Weight of Load (kg)</th>
<th>3.86</th>
<th>8.41</th>
<th>12.95</th>
<th>17.5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Laser Measurement (mm)</td>
<td>300.000</td>
<td>299.885</td>
<td>299.766</td>
<td>299.646</td>
</tr>
<tr>
<td>Displacement (mm)</td>
<td>0.115</td>
<td>0.119</td>
<td>0.120</td>
<td></td>
</tr>
</tbody>
</table>

The total vertical displacement that the composite link underwent was measured as 0.354 [mm]. However since the attachment bracket was bonded to the link with epoxy, the displacement due to adding the first 3.85 [kg] could not be measured and thus the aforementioned value was only due to three 10 [lb] weighted discs; a payload of 13.65 [kg]. Therefore the total deflection, $d_1$, that the link undergoes due to a payload of
17.5 [kg] was interpolated from the data shown in Table 4.1 using:

$$d_1 = w_1 \frac{\Delta d}{\Delta w}$$  \hspace{1cm} (4.1)

where $w_1$ is the weight of the attachment bracket and weight support, 3.85 [kg], $\Delta d$ is the total displacement measured, 0.354 [mm], due to a total weight, $\Delta w$, of 13.65 [kg]. Therefore, $d_1$ was determined to be approximately 0.100 [mm]. Therefore, the total displacement that the link undergoes due to the maximum payload of 17.5 [kg] for the prototype was assumed to be 0.454 [mm].

### 4.1.1.4 Analysis

The percentage error between the values obtained from the various tests using the same loading was less than 4%. Despite the fact that this indicates that the testing procedure is fairly consistent, there are a few sources of error present. First, the composite link underwent bending and thus had components of displacement in both the vertical and horizontal direction. The measurements taken by the laser only indicate the vertical component of displacement. Another source of error was the fact that the laser was not positioned at the very end of the link, only near it. The absolute end of the composite link would actually experience a displacement that is greater than 0.454 [mm]. Another source of error was the settling of the link. The laser data continued to vary as the link continued to creep. In order to eliminate this from affecting the results, the measurements were taken at the same time intervals for all 5 series of tests. This however still presented an issue as upon taking the load off of the link, the link did not return back to exactly 30 [cm] above the laser.

### 4.1.2 Repeatability Testing of the 2-DOF Prototype

As mentioned in Section 3.5, the 2-DOF main-arm prototype was constructed with a PR110 motor, a PR090 motor, and two identical carbon fiber-epoxy composite links.
The PR110 motor was bolted to the ground to secure the prototype. The same laser that was used in the previous test was secured to the ground. It was positioned such that the laser would shine near the end on the underside of the second link. Due to limitations of the motors with respect to their torque capabilities, the tests were performed with only a payload of the attachment bracket and weight support (3.85 [kg]).

4.1.2.1 Motor Control

The motors were connected in series to a PC using Computer Area Network (CAN) protocol to communicate between them and the PC. A simple program was used to move the two links in joint space using the PowerCubes’ built-in controllers.

4.1.2.2 Results

As illustrated in Figure 4.4, the first link was positioned 45° from vertical and the second link was positioned 45° from the first such that the longitudinal axis of the second link was parallel to the ground. Both motors were rotated at a speed of 10° per second, moving the links to 0° from vertical such that the arm was extended fully and pointing straight up. The links were then moved at the same speed back to the original position of (45°, 45°), and the laser measured the vertical distance to the link. This was repeated another four times to measure the repeatability.

The repeatability of the 2-DOF main-arm prototype was tested at a second location. The first link was positioned 55° from vertical and the second link was positioned 35° from the first as illustrated in Figure 4.5. As was the case with the previous position, the longitudinal axis of the second link was parallel with the ground. The same procedure was used to measure the vertical displacement of the link in order to test the repeatability. The results for both tests are shown in Table 4.2.

The results obtained from the repeatability tests shown in Table 4.2, indicate a high
level of repeatability for the proof-of-concept prototype. When the repeatability was tested at a location with both joint angles at 45°, the maximum variation between any two of the five measurements was 0.031 [mm]. For the case of the proof-of-concept prototype positioned with the PR110 motor and PR090 motor having joint angles (55°, 35°), the maximum variation between any two of the five measurements was 0.049 [mm]. This indicates that the proof-of-concept prototype has a repeatability of approximately ± 0.05 [mm].
Table 4.2: Laser Measurements For Repeatability Test

<table>
<thead>
<tr>
<th>Trial #</th>
<th>Measurement (mm) @ (45°, 45°)</th>
<th>Measurement (mm) @ (55°, 35°)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>35.460</td>
<td>45.868</td>
</tr>
<tr>
<td>2</td>
<td>35.491</td>
<td>45.828</td>
</tr>
<tr>
<td>3</td>
<td>35.481</td>
<td>45.850</td>
</tr>
<tr>
<td>4</td>
<td>35.470</td>
<td>45.843</td>
</tr>
<tr>
<td>5</td>
<td>35.472</td>
<td>45.819</td>
</tr>
</tbody>
</table>

4.1.3 Destructive Tests

Polymer composite materials are known to be considerably weak in the transverse direction and thus are prone to failure due to compressive forces. Therefore, two tests were performed on the carbon fiber-epoxy composite link to ensure that catastrophic failure does not occur when a compressive force is applied. The tests were performed using the LS100Plus materials testing machine by Lloyd Instruments to apply a continually increasing force to ensure that if the motion of the link were to ever be obstructed it would not lead to catastrophic failure.

4.1.3.1 Compression Test

A purely compressive test was performed on the composite link. Due to size limitations of the LS100Plus, a 65 [cm] long portion of the composite link was tested. An 11.25 [cm] section of the link was cut off on both ends using a vertical drop saw. The compressive plates on the LS100Plus have a circular shape with a diameter of 10 [cm]. Consequently, when placed next to the cross-section of the link, the link extends beyond the circular plate. Therefore, in order to apply the compressive force evenly across the link’s cross-section an adapter plate was created.

As shown in Figure 4.6, the adapter plate consists of a circular profile that matches up with the compressive plate and a rectangular profile that matches up with the link’s cross-section. A hollow steel cylinder with an inner diameter of approximately
10.5 [cm] was cut to a length of 5 [cm]. A 7.5 [mm] thick steel plate was cut into a rectangle that was 10 [cm] by 15 [cm]. The hollow cylinder was then centered on the plate and welded to it to create a rigid connection. Four 0.25 [mm] thick 10 [cm] by 2 [cm] strips of steel were then cut. The four strips were then welded on the opposite side of the rectangular plate such that they would surround the exterior profile of the prototype link. Strips of rubber were then glued on the inner side of the plates in order to help reduce the creation of stress concentrations as the compressive force was applied.

![Figure 4.6: Adapter Plate Used for Compression Test](image)

The section of composite link was loaded into the LS100Plus. The two adapter plates were placed between the link and the compressive plates as illustrated in Figure 4.7. The plate was lowered so that it was in slight contact with the adapter plate. The door was closed and the upper compressive plate was lowered at a constant speed applying an increasing force on the link. Once the force reached 98,412 [N], the link failed. The displacement of the top plate at this point was 4.5888 [mm]. The compressive plate continued to be lowered, compressing the link even further. However, the compressive force required was lower as indicated by the outliers in the graph shown in Figure 4.8. Therefore, this indicated that once the composite link did fail, the strength of the link was decreased but catastrophic failure did not occur.
Figure 4.7: Apparatus for Compressive Test

The section of the link after the compression test is illustrated in Figure 4.9. As can be seen, the link split at its edge at the location of buckling. This makes sense considering the way the link was produced. As mentioned in Section 3.4.4, the twill was not wrapped around the exterior profile but rather was cut into individual pieces for the four sides of the rectangular link. Therefore, the fibers are not continuous around the edges and thus a seam exists. As previously mentioned, the full-scale arm is designed to be produced using filament winding and thus this would not be an issue. Despite the fact that the prototype link was weakened due to this seam, it was still capable of withstanding a compressive force of 98,412 [N], which is equivalent to more than 10 metric tonnes.
Figure 4.8: Compressive Force Versus Displacement of Composite Link

Figure 4.9: Section of Link After Compressive Test
4.1.3.2 Three Point Bending Test

Since bending is the primary force that will be acting on the manipulator, it is also necessary to analyze the failure of the composite link due to bending. This was achieved using the LS100 Plus and the three point bending apparatus. An even amount was cut off on both ends of the composite link leaving a 45 [cm] long section. The three point bending apparatus was placed in the middle of the machine and the contacts were spread out to their maximum span of 32 [cm]. The section of the composite link was centered on the contacts vertically as shown in Figure 4.10. The third contact plate which was mounted to the driving actuator was lowered such that it was just touching the link.

![Apparatus for Three Point Bend Test](image)

Figure 4.10: Apparatus for Three Point Bend Test

The drive actuator was lowered at a constant speed applying a force in the middle of the link. The link fractured after a deflection of 30.707 [mm] due to a force of 7,518 [N]. As was the case with the previous test, after the link had fractured it remained intact. The force required to deflect the link afterwards, however, was decreased as illustrated by the outliers in the graph shown in Figure 4.11.
The force being applied in the middle of the section of link being tested caused it to bow down such that a tensile force was applied on the bottom portion and a compressive force on the top of the link. Figure 4.12, illustrates that the link fractured at the top. This makes sense since composite materials are considerably stronger in tension than in compression.

Beyond just fracturing, the composite link also delaminated. In the middle of the I-beam support, the two modular base units split apart from one another as they bowed in opposite directions during the bending test. The splitting of the two halves, however, only occurred near the area where the force was applied. Additionally, the top layer of exterior profile that surrounds the modular base units also delaminated near the force application area. This indicates that although the seam along which the two modular base units are in contact is an area of weakness, the epoxy bond between them is sufficient.
4.2 Finite Element Analysis

4.2.1 Procedure

The link was modeled as an assembly of parts. It consisted of two smaller rectangular parts that represented the modular base unit and one larger rectangular part that represented the exterior profile of the link as represented in Figures 4.13 and 4.14. Due to the modeling limitations of Abaqus pertaining to tapering, the parts were made in SolidWorks. The parts were then saved as an IGS file and imported into Abaqus.

These parts were designed to the same dimensions as the physical prototype. Additionally, the virtual parts were constructed as surfaces in order to accurately model the composite prototype. Therefore, the surfaces were defined to have the same dimensions as the exterior dimensions of the physical link.
4.2.2 Material Properties

The link was modeled as an elastic material of type lamina so that layering information of the link could be included. The mechanical properties of the material were defined to be the same as those used by Stanley in [47] for a uni-directional graphite fiber-epoxy composite. Therefore the longitudinal and transverse modulus of elasticity, $E_l$ and $E_t$, were defined as 135 [GPa] and 13 [GPa], respectively. The shear modulus, $G_{12}$, $G_{13}$, and $G_{23}$ (where the 1-direction is along the fibers, the 2-direction is transverse to the fibers in the laminate, and the 3-direction is normal to the laminate) were defined as 6.4 [GPa], 6.4 [GPa], and 4.3 [GPa]. The Poisson’s ratio, $\nu_{12}$, was defined as 0.38.
The virtual prototype was defined to have a density that was equal to that of the physical prototype. Since both the mass and volume of the prototype link are known, the density, $\rho$, is determined accordingly:

$$\rho = \frac{m}{V}$$  (4.2)

where $m$ is the mass and $V$ is the volume of the prototype link. Therefore, for a mass of $2.21$ [kg] and a volume of $0.00137326$ [m$^3$], the density is approximately $1,609$ [kg/m$^3$].

### 4.2.3 Modeling

The virtual prototype was created using conventional shell elements. In this, a layering pattern was defined for the virtual prototype that was identical to the layering pattern that was used for the production of the physical prototype. Therefore, the smaller rectangular surface that represents the modular base unit was defined as having a layering pattern of $0^\circ$, $45^\circ$, $0^\circ$, $0^\circ$, $45^\circ$, and $0^\circ$. The layering pattern for the larger rectangular surface that represents the exterior profile of the link was defined as $0^\circ$, $90^\circ$, $0^\circ$, $0^\circ$, $45^\circ$, $0^\circ$. Each layer was defined as having a thickness of $0.2833$ [mm] such that the total part thickness was $3.4$ [mm].

The surfaces shown in Figure 4.13 and 4.14 were assembled to create the completed virtual link. Two of the smaller rectangular surfaces were aligned such that they were parallel to each other and placed side by side with their side walls contacting each other. A tie constraint was utilized to bound the two touching faces together. The larger rectangular surface was also aligned parallel to the two smaller rectangular surfaces. It was positioned such that it surrounded the two smaller rectangular surfaces and was an equal distance away from all the exterior faces around its entire profile. Another four tie constraints were used to bind the four walls of the large rectangular surface to the parallel walls of the two smaller rectangular surfaces. This created a
single component by completely bounding the three individual surfaces together. In order to perform the FEA, a mesh was created for the virtual prototype. All the surfaces were constructed using the same mesh properties. The mesh was created using the free technique and the advancing front algorithm. The shape of the mesh elements were defined as being quad-dominated. The size of the elements was chosen to be 8 since this equated to the largest number of nodes that was less than the permitted 20,000 for the software.

### 4.2.4 Loading Conditions

In order to successfully model the prototype link, the faces on the virtual prototype link were partitioned. A rectangle was partitioned on all surfaces at both ends of the virtual link that had the same dimensions as the plate of the attachment bracket discussed in Section 3.5. An *encastré* boundary condition was created on this area on all six faces at the wider end of the link as shown in Figure 4.15. This represented a completely rigid joint between the attachment bracket and the link that permits no movement of the link relative to the attachment bracket.

![Figure 4.15: Bounded End of Virtual Prototype Composite Link](image)
Two loads were applied to the part. The first was a *gravity* load that represented the weight of the link itself. The second load, which was applied in the same direction of the gravity load, represented the weight of the payload. It was created as a *surface traction* load. As was the case with the boundary condition, it was applied on the area where the attachment bracket and the link were in contact as illustrated in Figure 4.16. The surface traction, $P_{ST}$ was determined as:

$$P_{ST} = \frac{Wg}{6A_{plate}}$$  (4.3)

where $A_{plate}$ is the area of one of the plates on the attachment bracket. Therefore, for a payload of 17.5 [kg] acting under the gravitational acceleration of 9.81 [m/s$^2$], the pressure acting on each 0.0090 [m$^2$] area that represents the plate was 3179.17 [Pa].

![Figure 4.16: Surface Traction on Virtual Prototype Composite Link](image)

**4.2.5 Results**

The virtual prototype was solved using the loading scenario described above. An exaggerated view of the link during loading is shown in Figure 4.17. It was determined that under a payload of 17.5 [kg], the carbon fiber-epoxy composite link deflected a maximum of 0.09195 [mm] at the end of the link where the load was applied.
In order to assess how well suited carbon fiber-epoxy composite is for this application, an aluminum and steel virtual prototype link were also tested. The links were constructed as a single member that has the same dimensions as the physical prototype. The identical loading scenario used for the composite link was utilized here. The steel link was defined as plain carbon steel alloy A36 with a density of 7,850 \( \text{[kg/m}^3 \text{]} \), modulus of elasticity of 207 \([\text{GPa}]\), and Poisson’s ratio of 0.3. The aluminum link was defined as aluminum alloy 6061 which has a density of 2,700 \([\text{kg/m}^3 \text{]} \), modulus of elasticity of 69 \([\text{GPa}]\), and Poisson’s ratio of 0.33 [5].

The results for the links constructed of the three different materials is shown in Table 4.3. The FEA results indicated that the steel link deflected the least. The composite link was second, deflecting about twice as much. The aluminum link was the least favourable, deflecting nearly three times as much as the steel link. However, the steel link weighed nearly five times more than the composite link. Therefore the composite link is about two and a half times better than the steel in terms of deflection relative to its weight.
Table 4.3: Comparison of Prototype Link Constructed of Different Materials

<table>
<thead>
<tr>
<th>Material</th>
<th>Maximum Deflection</th>
<th>Weight</th>
</tr>
</thead>
<tbody>
<tr>
<td>Composite</td>
<td>0.09195 mm</td>
<td>2.21 kg</td>
</tr>
<tr>
<td>Steel</td>
<td>0.04664 mm</td>
<td>10.78 kg</td>
</tr>
<tr>
<td>Aluminum</td>
<td>0.1225 mm</td>
<td>3.71 kg</td>
</tr>
</tbody>
</table>

### 4.3 Comparison of Physical and Virtual Prototype

The results from the single link static testing for the physical prototype were compared with those found for the virtual prototype. The deflection for the physical prototype was determined to be 0.454 [mm] for a payload of 17.5 [kg]. Meanwhile, the deflection for the virtual prototype was found to be 0.09195 [mm].

Despite the discrepancy between the results obtained for the physical and virtual prototype, the solution obtained for the virtual prototype is acceptable. The results for both tests are roughly the same considering that they are on the same order of magnitude. Additionally, the composite link was constructed by hand as mentioned in Section 3.4.1. As a result, the constructed links have various flaws and imperfections. Also, the strength of the link was compromised due to the production method utilized. The carbon fiber material was not a continuous piece; rather it was cut into sections for each face. As a result, a seam existed on the corners. As mentioned in Section 4.1.3, this seam was the point of failure for the link. In addition, the prototype links were not cured in an autoclave which reduced its strength. It is expected that links manufactured using filament winding and cured in an autoclave would have deflections closer to the FEA results.

### 4.4 Summary

Static testing was performed on the carbon fiber-epoxy composite link. The results indicated that the composite link effectively limited the amount of deflection expe-
rienced at the end of the link. Repeatability testing that was performed on the 2-DOF main-arm proof-of-concept prototype illustrated the damping capabilities of composite materials by providing a repeatability of approximately $\pm 0.05 \text{[mm]}$. Two destructive tests showed that catastrophic failure did not occur and that carbon fiber-epoxy composite’s weakness in compression is not an issue for this application. The results from the FEA on the virtual prototype indicated that the theoretical deflection was less than that found for the physical prototype link. However, the agreement between the two values was enough to indicate that if the links were to be constructed using filament winding and cured in an autoclave the composite links could provide considerably high stiffness.
Chapter 5

Design of the Full-Scale Robotic Arm

In this chapter the design of the full-scale arm is discussed. Suitable motors and gearheads were selected for the operation of joints two and three. Static testing was performed on the part in order to determine the deflection when the links are extended straight out. Links of varying dimensions were tested. The optimal dimensions and layering pattern for minimal weight for a link that has stiffness equivalent to industry standards was determined.

5.1 Motor Selection

In order to accurately test the static deflection of the two main links when extended outwards, the motor that will operate the second main link (i.e., joint 3) must be identified. The speed of the motor was selected by observing speeds of motors used in other large-scale robotic arms. Table 1.1 indicates that the Kuka KR 100-2P and Motoman EPH100, the two largest arms, have a speed of 95 °/s and 110 °/s, respectively. However, the reach of these arms is only 3.5 [m] and 3.0 [m]. Since the robotic arm to be designed has a longer reach, the speed of the motor should be less.
Therefore, the motor speed, $\omega$, was selected to be 60 [°/s]. The selected motor must be capable of permitting the robotic arm to meet the specifications outlined in Section 1.2. Consequently, it must enable the robotic arm with a reach of 5 [m] and payload of 50 [kg] to move the tool at a linear speed of at least 0.15 [m/s]. Assuming that the motor that operates the first main link has a speed of at least 30 [°/s], half the speed of the motor that operates the second link, then the tool has a linear speed of at least 1.25 [m/s]. This scenario is demonstrated in Figure 5.1. It illustrates the operation of the motors for two seconds. Using basic trigonometry, $X$ is determined to be 1.25 [m]. Therefore, the distance between the start of the first link and end of the second link is 2.5 [m]. Consequently, the arm undergoes a linear displacement of 2.5 [m] during a two second interval and thus it has a linear speed of 1.25 [m/s].

![Figure 5.1: Scenario for Motion of Full-Scale Robotic Arm](image)

The power necessary to move the second link and full payload at the desired speed must be calculated in order to select an appropriate motor. The free body diagram for the loading scenario is illustrated in Figure 5.2. In order for static equilibrium, the torque provided by the motor, $M_{\text{motor}}$, is:

$$M_{\text{motor}} = W_{\text{Link2}} \frac{L_{\text{Link2}}}{2} + W_{\text{EE}} L_{\text{Link2}}$$

(5.1)

where $L_{\text{Link2}}$ is the length of the second main link, $W_{\text{Link2}}$ is the weight of the second
main link, and $W_{EE}$ is the weight of the end-effector. It was assumed that $L_{Link2}$ was 2.5 [m] since the two main links provide the majority of the 5 [m] reach and are the same length, the mass of the link was 250 [kg], and the mass of the end-effector was 80 [kg] (50 [kg] payload plus 30 [kg] wrist). Therefore, the required moment of the motor is determined to be 5,027.6 [Nm].

![Free Body Diagram of Second Main Link](image)

Figure 5.2: Free Body Diagram of Second Main Link

The required power, $P$, of the motor is defined as:

$$ P = M_{motor} \omega \frac{2\pi}{360^\circ} \quad (5.2) $$

Therefore, in order to provide the necessary moment of 5,027.6 [Nm] at a desired speed of 60 [°/s], the motor is required to have a power of 5,264.9 [W].

Rotary servomotors from Danaher Motion were investigated in order to determine a suitable motor. In doing so, a safety factor of two was included such that the motor was required to have a power of 10,529.8 [W]. Therefore, Model B-804-A which is capable of 10,800 [W] was chosen (See Appendix B.1) [48].

A suitable gearhead for operation with the motor was also selected. The required gearhead ratio, $G.R.$, is calculated as:

$$ G.R. = \frac{RPM_{motor}}{\omega \frac{60s}{\min} \frac{rot}{360^\circ}} \quad (5.3) $$
where \( RPM_{motor} \) is the speed of the motor in rotations per minute. Since the motor runs at 1,500 [rpm], a gearhead ratio of 150:1 is required to slow the speed to the desired 60 [°/s]. Therefore, gearhead DTR90-150 was selected because it has a ratio of 150:1 and its continuous rated torque at 3,000 [rpm] for 10,000 hour life of 81 [Nm] is slightly greater than the continuous stall torque of the B-804-A motor of 78.6 [Nm] operating at full speed of 1,500 [rpm] (See Appendix B) [48].

5.2 Dimensional Analysis of Links

5.2.1 Purpose
The full-scale robotic arm is to be designed such that it has a stiffness that is equivalent or better than other robotic arms currently available. In a study performed by Airbus, four robotic arms were fully loaded and tested in their theoretical worst position for deflection. The results of the static flexibility determined from the test is shown in Table 5.1 [49]. The Kuka KR 60-3 has the lowest static flexibility of 2.0 [mm]. However, this arm only has a payload and reach of 60 [kg] and 2,033 [mm]. Since the full-scale robotic arm is intended to have a payload of 50 [kg] and reach of 5 [m], the robotic arms rated for larger payloads provide more comparable data. The Kuka KR 240-2 and ABB IRB 7600 have a static flexibility of 3.4 [mm] and 3.5 [mm], respectively. Therefore, the full-scale arm should have a static flexibility that is no worse than 3.4 [mm].

FEA was performed on the two main links of the full-scale robotic arm since they provide the majority of the reach for the robot. Therefore, the two main links were to be designed such that they met the static flexibility requirement mentioned above. Since only the two main links are being examined, the static flexibility of the base, motors, and wrist configuration were not included. Therefore, the static flexibility of the two main links was designed to be less than 3.4 [mm]. Since the two main links do
provide the majority of the reach to the robot, their contribution to the total static deflection is significant. Thus, designing the two main links to have a static deflection of 2.0 [mm] should ensure that the total static flexibility is no more than 3.4 [mm] considering that the motors are very compact and undergo very little deflection and that the hysteresis of the motors is insignificant [49].

Table 5.1: Static Flexibility of Four Commercial Robotic Arms

<table>
<thead>
<tr>
<th></th>
<th>Payload (kg)</th>
<th>Max Reach (mm)</th>
<th>Static Flexibility (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Kuka KR 60-3</td>
<td>60</td>
<td>2033</td>
<td>2.0</td>
</tr>
<tr>
<td>Staubli RX170</td>
<td>65</td>
<td>1835</td>
<td>7.0</td>
</tr>
<tr>
<td>Kuka KR 240-2</td>
<td>240</td>
<td>2701</td>
<td>3.4</td>
</tr>
<tr>
<td>ABB IRB 7600</td>
<td>400</td>
<td>2550</td>
<td>3.5</td>
</tr>
</tbody>
</table>

Considering that the full-scale robotic arm will be constructed using filament winding, a virtual model in Abaqus should provide useful results. FEA was used to determine the static flexibility of the two main links. It was used to test a variety of dimensions for the links in order to achieve the desired static flexibility of 2.0 [mm]. In doing so, the dimensions of the links were chosen such that the weight of the links was minimized.

5.2.2 Testing Procedure

The static flexibility was determined for the worst position for deflection, the arm extended straight out horizontally. Therefore, the two main links were modeled as a single component. The attachment to the base link was represented by bounding all faces of the initial 20 [cm] of the part as illustrated in Figure 5.3. The *encastré* boundary condition was used to fully restrict this portion of the part.

As was the case for the FEA performed on the prototype, the payload of 80 [kg] was represented by a *surface traction* load that was acting on the six vertical faces of the link directed in the same direction of gravity. This force was applied on the final 20 [cm] section of the part. A *gravity* load was also applied to represent the weight of the
links. Since the links are to be made using filament winding, the density was assumed to be 1,700 [kg/m$^3$], equal to high modulus carbon fiber-epoxy composite [5].

A load was also used to represent the weight of the motor and gearhead for joint three. The motor and gearhead have a mass of 50.6 [kg] and 5.5 [kg], respectively [48]. Therefore, a mass of 60 [kg] was used to represent their weight and the attachment devices to the two main links. Additionally, this force was applied in the middle of the length of the model since it is assumed that the two main links have an equal length. The weight was presented as a pressure. The force was applied on an area that represented the size of the motor. As shown in Appendix B.1, the base of the motor is approximately 45 [cm] by 20 [cm]. Therefore, the pressure was applied on this same area.

![Figure 5.3: Loading Scenario for Model of Full-Scale Main Links](image)

### 5.2.3 Results

#### 5.2.3.1 Links with Height Twice the Width

The first design of component that represents the two main links (Design A) was created with height, width, and wall thickness of 400 [mm], 200 [mm], and 3.4 [mm], respectively. Based on the results from Section 2.2.3, it was designed with a constant
wall thickness and a negative 0.5° taper for the height and width. The aforementioned values for height and width are those at the broad end of the part. The part was designed with the same layering pattern that was used for the prototype; 0°, 45°, 0°, 0°, 45°, 0°, 0°, 90°, 0°, 0°, 45°, 0°. Additionally, the edges of the component were rounded with a radius of 10 [mm]. This was included because it helps reduce stress concentrations resulting in a stronger link. The radius also helps aid in the removal of the mold during production.

The results from the FEA for the main links with the aforementioned dimensions revealed that the maximum static deflection was 4.804 [mm]. This is considerably larger than the desired value of 2.0 [mm]. Therefore, the link was redesigned. Since Design A had a static deflection that was more than two times as much as desired, the link was redesigned to be as large as possible for Design B. As mentioned in Section 1.2, the links of the robotic arm must be capable of fitting through a 0.76 [m] diameter opening. Therefore, the model representing the two main links was designed to be as large as possible while still satisfying this requirement. The broad end was designed to span 0.74 [m] between opposite corners such that 1 [cm] of clearance is provided at all corners as illustrated in Figure 5.4. Using the same design constraint that the height is twice the width, basic trigonometry reveals that the angle θ is equal to tan⁻¹ 2. Therefore, the maximum height and width that can fit through the opening are 0.662 [m] and 0.331 [m], respectively.

The results from the FEA in Abaqus for Design B revealed that the links were over-engineered, resulting in a maximum static deflection of 1.167 [mm]. Therefore, in order to minimize the weight, the size of the links were designed to be smaller. The height and width of the links for Design C were modified to be smaller than Design B and larger than Design A. Considering the static deflection for Design B was closer to the desired value than Design A, the link was redesigned to have dimensions closer to the Design B than Design A. Therefore, the height and width at the broad end
of the part representing the two main links was defined as 600 [mm] and 300 [mm], respectively. FEA determined that the static deflection was 1.499 [mm]. Therefore, it too was over-engineered.

In order to determine the dimensions for the links that result in a static deflection of 2.0 [mm], the results for Designs A, B, and C were plotted as illustrated in Figure 5.5. The static deflection of the link was plotted against its average cross-sectional area. The results revealed that a non-linear relationship exists between the two variables. According to the trend line mapping the points, a link with a cross-sectional area of 6,500 \([\text{mm}^2]\) has a static deflection of 2.0 [mm] for the defined loading scenario. Therefore, the link was redesigned such that Design D has a cross-sectional area of 6,500 \([\text{mm}^2]\). Assuming a negative taper angle of 0.5° and that the height is twice the width, Equation (2.8) for the cross-sectional area of the I-beam supported rectangular structure is rearranged for the width, \(w\), as:

\[
w = \frac{2A_{c,\text{avg}} + 436.35t + 12t^2}{16t}
\]  

(5.4)

Therefore, assuming a wall thickness of 3.4 [mm], the width of the link should be
approximately 269 [mm] for a link with an average cross-sectional area of 6,500 [mm$^2$]. Consequently, the height should be 538 [mm].

Figure 5.5: Average Cross-Sectional Area Versus Static Deflection for Links with Width-to-Height Ratio of 2.0

The FEA procedure revealed that Design D had a static deflection of 2.054 [mm]. This is sufficiently close to the desired value of 2.0 [mm].

5.2.3.2 Relationship of Thickness and Static Deflection

The four previous links that were tested all had a wall thickness of 3.4 [mm]. Therefore, a link with a different wall thickness was tested to determine the relationship between the thickness and the static deflection.

The modified link, Design E, was defined to have a wall thickness of 5.1 [mm], one and a half times larger than that of the previous links. In order to effectively test how the variance of the wall thickness affected the static deflection, the other variables were held constant. Therefore, the link was designed to have a cross-sectional area...
of 6,500 [mm$^2$] so that it is the same as Design D. Using Equation (5.4), the width at the broad end of the part was determined to be 190.5 [mm] in order for the link with 5.1 [mm] thick walls to have a cross-section equal to Design D. Consequently, the required height of the link was 381 [mm].

The thicker link, Design E, was tested using the same procedure for FEA and it was determined that it has a static deflection of 4.163 [mm]. Its static deflection was considerably larger than the link with thinner walls. In fact, a 50% increase in the wall thickness led to an increase in the static deflection of more than 100%. This agrees with the results found for the optimization of the prototype link as discussed in Section 2.3.9.4.

5.2.3.3 Links with Optimized Height-to-Width Ratio

According to the results found in Section 2.3.9.4, the static deflection of the I-beam supported rectangular links was minimized when the height was 3.63 times as large as the width. Therefore, the model that represents the two main links was designed to this constraint.

For a link with a negative taper angle of 0.5° and a height-to-width ratio of 3.63, the width is defined as:

\[ w = \frac{2A_{c, avg} + 436.35t + 12t^2}{25.78t} \]  

(5.5)

In order to determine how the variation of the height-to-width ratio affects the static deflection, the model was designed to have a cross-sectional area and wall thickness equal to that of the model with a height twice its width (Design D). Therefore, for a cross-sectional area of 6,500 [mm$^2$] and wall thickness of 3.4 [mm], the width at the broad end of the link was 167 [mm]. Consequently, for a height-to-width ratio of 3.63, the height of the link was 606 [mm].

The same FEA procedure was performed on Design F of the links and it was de-
terminated that it had a static deflection of 1.859 [mm]. This is a lower value than that found for the link with a height-to-width ratio of 2. Considering that the two components were designed to have the same average cross-sectional area, links with a height-to-width ratio of 3.63 do indeed have a higher stiffness-to-weight ratio than those with a with a height-to-width ratio of 2.

The static deflection determined for Design F was however smaller than the desired 2.0 [mm], indicating that it was over-engineered. Therefore, in order to reduce the weight of the links, the dimensions of the links were reduced for Design G.

The height at the broad end of the link was defined to be 538 [mm], such that it was equal to that for the link with a height-to-width ratio of 2. For a width-to-height ratio of 3.63, the width of the link was defined to be 148 [mm]. The results from the FEA for Design G revealed that it had a static deflection that was too high at 2.650 [mm].

In order to achieve a static deflection of 2.0 [mm], the link was redesigned to be larger than the Design G in order to improve its stiffness. The height and width of the link were chosen to be slightly smaller than that of the original link that had a height-to-width ratio of 3.63 (Design F). Therefore, the height and width were defined to be 581 [mm] and 160 [mm] for Design H. It was determined that it had a static deflection of 2.106 [mm], slightly higher than the desired 2.0 [mm].

5.2.3.4 Different Layering Patterns

One of the major advantages of composite materials is that since their mechanical properties are directional, they vary widely. Therefore, the stiffness of the links can be improved by varying the layering pattern. Since the deflection of Design H for the model representing the two main links is slightly above the desired value, alternate layering patterns were investigated to reduce the static deflection.

The arm ratio as defined by Equation (3.1), is approximately 9.3 for Design H. Ac-
According to Figure 1.8, approximately 19% of the layers should be oriented at 45° to the longitudinal axis. Therefore, 2 of the 12 layers should be oriented in this direction. Also, it was mentioned in Section 3.4.3 that one 90° layer should be included to help reduce the hoop stress that exists at the joints. Therefore, the remaining 9 layers were designed to be 0°.

Numerous different layering patterns with a repetitive order were investigated for the links. The results of eight such layering patterns are shown in Table 5.2. The results indicate that including one +45° layer and one -45° layer helps to improve both the strength and stiffness of the links. Also, the results indicated that winding the layers at a -45° angle before winding them at a +45° angle leads to a reduction in the stress. Additionally, it was determined that it is best to have the 90° layer in between the positive and negative 45° layers. The optimal fiber layer for reducing both the stress and deflection was determined to be -45°, 0°, 0°, 0°, 0°, 0°, 90°, 0°, 0°, 0°, 0°, 45°.

Table 5.2: Stress and Deflection in Link for Different Layering Patterns

<table>
<thead>
<tr>
<th>Layering Pattern</th>
<th>Von Mises Stress (MPa)</th>
<th>Deflection (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0, 0, 0, 45, 0, 0, 90, 0, 0, 45, 0, 0</td>
<td>12.59</td>
<td>1.913</td>
</tr>
<tr>
<td>0, 0, 0, -45, 0, 0, 90, 0, 0, -45, 0, 0</td>
<td>12.59</td>
<td>1.912</td>
</tr>
<tr>
<td>0, 0, 0, 45, 0, 0, 90, 0, 0, -45, 0, 0</td>
<td>11.28</td>
<td>1.892</td>
</tr>
<tr>
<td>0, 0, 0, -45, 0, 0, 90, 0, 0, 45, 0, 0</td>
<td>11.23</td>
<td>1.892</td>
</tr>
<tr>
<td>0, 0, 0, -45, 0, 0, 45, 0, 0, 90, 0, 0</td>
<td>11.30</td>
<td>1.893</td>
</tr>
<tr>
<td>0, 0, 0, 90, 0, 0, -45, 0, 0, 45, 0, 0</td>
<td>11.58</td>
<td>1.916</td>
</tr>
<tr>
<td>0, 0, 0, 0, 0, -45, 90, 45, 0, 0, 0, 0</td>
<td>12.61</td>
<td>1.913</td>
</tr>
<tr>
<td>-45, 0, 0, 0, 0, 0, 90, 0, 0, 0, 0, 45</td>
<td>10.87</td>
<td>1.892</td>
</tr>
</tbody>
</table>

Altering the layering pattern resulted in a reduction of the static deflection of 0.214 [mm]. This resulted in a link that was over-engineered with a static deflection of 1.892 [mm]. Therefore, in order to achieve a link that has the optimal layering pattern and a static deflection of approximately 2.0 [mm], the link was designed to have a static deflection of 2.214 [mm] for the original layering pattern.

The cross-sectional area was plotted against the static deflection for the three models.
with a height-to-width ratio of 3.63 and original layering pattern (Designs F, G, and H). As was the case for the links with a height-to-width ratio of 2, the cross-sectional area is non-linearly related to the static deflection. Figure 5.6 illustrates that a link with an average cross-sectional area of 6,075 $[\text{mm}^2]$ has a static deflection of 2.214 $[\text{mm}]$.

![Figure 5.6: Average Cross-Sectional Area Versus Static Deflection for Links with Width-to-Height Ratio of 3.63](image)

The model was redesigned to have an average cross-sectional area of 6,075 $[\text{mm}^2]$ for Design I. Therefore, using Equation (5.5) the width was determined to be 157 $[\text{mm}]$. Therefore, the height was 570 $[\text{mm}]$ such that the height-to-width ratio is 3.63. The results from the FEA determined that the link with the original layering pattern had a static deflection of 2.227 $[\text{mm}]$. However, for a layering pattern of $-45^\circ$, $0^\circ$, $0^\circ$, $0^\circ$, $0^\circ$, $0^\circ$, $0^\circ$, $0^\circ$, $0^\circ$, $90^\circ$, $0^\circ$, $0^\circ$, $0^\circ$, the static deflection was 2.000 $[\text{mm}]$. 

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5.3 Discussion

The links were designed to have a static deflection of 2.0 [mm] while minimizing the average cross-sectional area to 6,075 \([\text{mm}^2]\) by designing the links to have a layering pattern of \(-45^\circ, 0^\circ, 0^\circ, 0^\circ, 0^\circ, 90^\circ, 0^\circ, 0^\circ, 0^\circ, 45^\circ\), a constant wall thickness of 3.4 [mm], a negative taper angle of 0.5\(^\circ\), and a height and width at the broad end of the first link of 570 [mm] and 157 [mm].

The mass of the two links can be calculated by rearranging Equation (4.2). The volume of the member representing the links is defined as:

\[
V = A_{c,\text{avg}}L
\]  

Therefore, for a length of 5 [m], an average cross-sectional area of 6,075 [mm\(^2\)], and a density of 1,700 [kg/m\(^3\)], the mass of the main links, \(m_{\text{mainlinks}}\), is only 51.64 [kg]. Therefore, each link weighs only 25.82 [kg]. Using Equation (5.1) and (5.2), the torque and required power of the motor at joint three was re-calculated as 2,278.6 [Nm], and 2,386.2 [W], respectively. In order to provide a safety factor of two, the motor should be capable of providing 4,772.3 [W]. Therefore, Model B-404-C which is capable of 5,400 [W] was selected (See Appendix B.1).

Since the motor operates at a speed of 5,000 [rpm], in order to slow the speed down to 60 [°/s] the gearhead ratio that is required was determined to be 500:1 using Equation (5.3). Gearhead DTR60-500 was chosen because it has a ratio of 500:1 and its continuous rated torque at 5,000 [rpm] for 10,000 hour life is 18 [Nm], whereas the continuous stall torque of the B-404-C motor is 13.1 [Nm] while operating at full speed of 5,000 [rpm] (See Appendix B).

The free body diagram for the loading scenario for the motor that operates the first main link (joint 2) is illustrated in Figure 5.7. In order for static equilibrium, the torque provided by the second motor, \(M_{\text{motor2}}\), is:
\[ M_{\text{motor}2} = L_{\text{Link}} \left[ \frac{W_{\text{Link}1}}{2} + W_{\text{Joint3}} + \frac{3W_{\text{Link}2}}{2} + 2W_{EE} \right] \] (5.7)

where \( L_{\text{Link}} \) is the length of the first main link (which is also the length of the second main link), \( W_{\text{Link}1} \) is the weight of the first main link, and \( W_{\text{Joint3}} \) is the weight of the motor and gearhead at joint 3. The mass of joint three is assumed to be 20 [kg] since the motor is 12.5 [kg] and gearhead is 2.7 [kg] and an attachment bracket is also necessary. Therefore, with \( L_{\text{Link}} \) equal to 2.5 [m] and \( W_{\text{Link}1} \) and \( W_{\text{Link}2} \) both 25.82 [kg], the required torque of motor two is determined to be 5,681 [Nm]. Using Equation (5.2) and assuming a speed of 60 [°/s], the required power of the motor is 5,949.1 [W]. Therefore, including a factor of safety of two, the motor must be capable of 11,898.2 [W]. Therefore, motor B-804-B which weighs 50.6 [kg] was selected since it can provide 13,900 [W] of power. Since it operates at 2,000 [rpm] and a continuous stall torque of 78.6 [Nm], gearhead DTR90-200 which weighs 5.5 [kg] and has a gear ratio of 200:1 was chosen since it can slow the motor down to 60 [°/s]. Furthermore, its continuous rated torque at 3,000 [rpm] for 10,000 hour life is 88 [Nm] (See Appendix B).

![Figure 5.7: Free Body Diagram of Second Main Link](image)

The total mass, \( m_{\text{total}} \), of the robotic arm is defined as:

\[ m_{\text{total}} = m_{\text{base}} + m_{\text{main links}} + m_2 + m_3 + m_{\text{wrist}} \] (5.8)
where \( m_{\text{base}} \) is the mass of the base joint and first motor, \( m_2 \) is the mass of joint two, \( m_3 \) is the mass of joint three, and \( m_{\text{wrist}} \) is the mass of the wrist configuration. Therefore, with \( m_3 \) weighing 60 [kg] assuming that the weight of the attachment bracket is small, \( m_2 \) weighing 20 [kg], \( m_{\text{mainlinks}} \) equal to 51.64 [kg], and \( m_{\text{wrist}} \) equal to 30 [kg], the mass of the robotic arm is 484.92 [kg] assuming that \( m_{\text{base}} \) represents two thirds of the total mass of the robotic arm.

By comparing the calculated mass of the full-scale robotic arm to those listed in Table 1.1, it is clear that the use of carbon fiber-epoxy composite provides a dramatic reduction in weight. The calculated mass is similar to the mass for robotic arms that have a payload of around 50 [kg] and reach of 2 [m]. However, the full-scale composite arm has the same payload but a reach of 5 [m]. This indicates that the robotic arm was successfully designed for high stiffness-to-weight ratio in order to minimize the total weight.

5.4 Summary

In this chapter the full-scale arm was designed with the use of Abaqus. It was determined that constructing the links with a height-to-width ratio of 3.63 improves the stiffness-to-weight ratio over those with a height-to-width ratio of 2.0. Additionally, it was determined that minimizing the thickness does improve the stiffness-to-weight ratio. Therefore the height and width at the broad end was chosen to be 570 [mm], 157 [mm], respectively. Additionally, the link was chosen to include a negative taper angle of 0.5° and constant wall thickness of 3.4 [mm]. Using a layering pattern of -45°, 0°, 0°, 0°, 0°, 0°, 90°, 0°, 0°, 0°, 0°, 0°, 45°, a static deflection of 2.000 [mm] was achieved while minimizing the weight of the two main links to 51.64 [kg].
Chapter 6

Conclusions and Recommendations
for Future Work

6.1 Conclusions

This thesis outlines the design of a novel 6-DOF robotic arm made of carbon fiber-epoxy composite material. This is envisioned to present a significant contribution to the robotic industry because even though the use of composite materials for robotic applications has been investigated over the last 20 years, a 6-DOF lightweight, long-reach robotic arm that is constructed primarily of a polymer composite material has not been designed as of yet.

A 6-DOF robotic arm with a payload of 50 [kg] and reach of 5 [m] was designed to be made of a polymeric composite material because it can help alleviate the limitations in terms of weight, speed, and repeatability of large-scale steel and aluminum robotic arms. Carbon fiber was selected as the support material because it is favourable over other alternatives in terms of stiffness due to its high tensile modulus and low ultimate tensile strain. The arm was designed to be constructed using long-strand carbon fibers because they provide further improvements as discussed in Chapter 1.
The robotic arm was designed to have a serial configuration with a kinematically-simple layout. This design provides the best opportunity for creating a lightweight robotic arm since it does not have any unnecessary offsets or link lengths between joints [38]. This is best-suited for designing the links with carbon fiber-epoxy composite since it is an expensive material. The main-arm configuration that was selected for the 6-DOF robotic arm was \( R \perp R \parallel R \), where the first revolute joint is perpendicular to the second revolute joint that is parallel to the third revolute joint. This layout is beneficial because revolute joints require links with less material than prismatic joints. Additionally, this layout has a very large workspace and minimal torsional forces acting on it which make it possible to construct a robotic arm to the required specifications while keeping the weight to a minimum.

The use of an internal support structure in the design of the links for the robotic arm can offer improvements in the stiffness-to-weight ratio. Therefore, the links were designed to have an I-beam supported rectangular cross-section in order to produce a lightweight robotic arm with the desired stiffness.

A 35% size prototype of the main-arm of the 6-DOF robotic arm was constructed. The design focused on the second and third links of the arm because they provide the majority of the reach for the robotic arm with the selected manipulator layout. Optimization was performed on these two I-beam supported rectangular linkages of the prototype. It was determined that for minimal deflection the thickness should be minimized and a height-to-weight ratio of 3.63 should be used.

The 35% size main-arm proof-of-concept prototype was constructed for testing purposes. One of the biggest contributions of this thesis work was the creation of a novel production process. This entails producing a series of hollow modular base units, ideally by filament winding (but for reasons of economics by hand lay-up for the construction of the prototype), and then winding around the entire series to create the part with an internal support structure. Using this method, two identical small
rectangular base units were first constructed. They were then bonded together and exterior layers of carbon fiber were wrapped around them. This technique proved to be very efficient.

Various tests were performed on the carbon fiber-epoxy composite prototype links. Destructive testing indicated that under normal operation the links could easily withstand the applied forces. When the applied forces were increased such that fracture of the link occurred it was determined that the composite material still maintains a portion of its strength and the links are able to retain their form to prevent catastrophic failure.

A 2-DOF arm was created using the prototype links that were constructed. The arm was operated and it was determined that the high damping capabilities of carbon fiber-epoxy composite enabled a high repeatability.

The static deflection of the prototype composite link was tested. It was determined that under full load, the link underwent a maximum deflection of 0.454 [mm]. The results were verified by creating a virtual model of the link and performing FEA using Abaqus software. The analysis determined that the maximum deflection was 0.09195 [mm]. The difference in deflections is due to problems encountered during the hand lay-up production of the links.

The full-scale robotic arm is intended to be produced using filament winding. Since this is an automated process it reduces the amount of human error that exists. Therefore, the links for the full-scale arm were designed using Abaqus software. Links of varying dimensions were tested in order to provide a total static deflection of the links of 2 [mm] while minimizing the weight. It was determined that for minimal weight, the links should be constructed with a constant wall thickness of 3.4 [mm], a negative taper angle of 0.5°, and a height and width at the broad end of the first main link of 570 [mm] and 157 [mm], respectively, such that the height-to-width ratio is 3.63. Additionally, it was determined that for maximum strength and stiffness the link should
be constructed using a layering pattern of $-45^\circ, 0^\circ, 0^\circ, 0^\circ, 0^\circ, 0^\circ, 90^\circ, 0^\circ, 0^\circ, 0^\circ, 0^\circ, 45^\circ$.

The combined weight of the two main links for minimal weight was determined to be 51.64 [kg]. Therefore, constructing the 6-DOF robotic arm out of carbon fiber-epoxy composite leads to a robotic arm with a total weight of approximately 485 [kg]. This weight is comparative to commercial robotic arms that have a payload of 50 [kg] and a reach of 2 [m]. Therefore, the use of carbon fiber-epoxy composite material enables an increase in the reach of 3 [m] without increasing the total weight when compared to current metal arms.

### 6.2 Recommendations for Future Work

The 6-DOF robotic arm is intended to be used for operation on airplanes and other large structures. Therefore, a method for moving the entire robotic arm needs to be investigated. For this reason, it would be beneficial to investigate the feasibility of using carbon fiber-epoxy composite for the production of the base link and wrist configuration since the work outlined in this thesis indicated that using this composite material for the production of the two main links resulted in dramatic weight reductions.

The motor selection that was performed was used only as a guideline so that the full-scale robotic arm could be modeled in Abaqus. In-depth investigation of motors needs to be performed in order to select appropriate motors for operation of all the links. Simultaneously, FEA needs to be performed on the virtual model of the robotic arm in order to design the links such that the robotic arm has the desired stiffness under these new loading conditions.

One of the major obstacles to the use of polymer composite materials in robotic applications is the difficulty in joining the motors to the links. Therefore, research needs
to be performed in joining techniques. Since drilling composite material leads to a re-
duction in their strength, non-destructive joining techniques need to be investigated.
One such method is using a carbon fiber-epoxy composite joint between the link and
the motor. This idea shows great potential because it permits the possibility of cre-
ating the link around the motor such that is essentially one unified part. However,
further research needs to be done pertaining to the design and production procedure.
Finally, after the completion of the above, a full-scale prototype of the arm needs to
be built. Extensive testing of the full-scale prototype must be conducted to ensure
the design is safe and robust for industrial applications.
References


Appendix A

MATLAB Code

A.1 Declaration of Constants

%Program: constants.m
%Declares constants that are necessary to perform calculations needed in determining
the optimal cross-section for minimal deflection.

function [L, g, rho, m_motor, W, M_max, eff, v, E] = constants

L=1.75; %Total length (m) of the two main links.
g=9.81; %Gravitational constant (m/s^2).
rho=1700; %Density (kg/m^3) of material that links are made of.
m_motor=4.7; %Mass (kg) of motor to operate 2nd main link.
W=17.5; %Payload (kg) to be supported by robotic arm.
M_max=425; %Maximum torque (Nm) of motor operating 1st main link.
eff=0.90; %Efficiency () of motor operating 1st main link.
v=0.25; %Poisson’s ratio () of material that links are made of.
E=220e9; %Elastic modulus (Pa) of material that links are made of.
A.2 Objective Function

%Program: objfun.m

%Determines the end-effector deflection of a rectangular cross-section $R \perp R \parallel R$ robotic arm.

function deflection = objfun(x)

%User inputs. Height, width, and thickness of rectangular cross-section.
  h=x(1);
  w=x(2);
  t=x(3);

%Input: m.file for determining cross-sectional area and moment of inertia for specific cross-sectional geometry.
%Output: cross-sectional area and moment of inertia of the links.
  [Ac] = A_I_beam_sup(h, w, t);
  [I] = I_I_beam_sup(h, w, t);

%Input: m.file containing constants of robotic arm under examination.
%Output: values of constants necessary to perform the following calculations.
  [L, g, rho, m_motor, W, M_max, eff, v, E] = constants;

  L_wrist=L/10; %Length (m) of wrist of robotic arm.
  G=E/(2*(1+v)); %Shear modulus (Pa) of material that links are made of.
  T=g*W*L_wrist; %Internal torque (Nm) acting on links at wrist location.
  %Thickness (m) of a hollow rectangular cross-sectional with an equal cross-sectional area to that of the link being examined.
  t_equiv=(2*(w+h)-sqrt((2*(w+h))^2-16*Ac))/8;
  %Angle (radians) of twist of the two main links.
\[ \phi = \frac{(T*L*(2*(w-t_{equiv})+2*(h-t_{equiv}))}{(4*G*((w-t_{equiv})*(h-t_{equiv}))^{2}*t_{equiv})}; \]

%Displacement as a result of twisting of main links due to torsional loading.
\[ dy = L_{wrist} \sin(\phi) \cos(\phi); \]

%Displacement due to bending of the two main links.
\[ v_{mainlinks} = \frac{(g* \rho * A_c * L^4)}{(8*E*I)} + \frac{(5*g*m_{motor}*L^3)}{(48*E*I)} + \frac{(g*W*L^3)}{(3*E*I)}; \]

%Displacement due to bending of wrist.
\[ v_{wrist} = \frac{(g*W*L_{wrist}^3)}{(3*E*I)}; \]

%Total deflection experienced by the 'end-effector'.
\[ \text{deflection} = v_{mainlinks} + v_{wrist} + dy; \]

### A.3 Non Linear Constraint

%Program: nonlincon.m

%Declares the non-linear inequality and equality constraints.

function [c, ceq] = nonlincon(x)

%User inputs. Height, width, and thickness of rectangular cross-section.
\[ h = x(1); \]
\[ w = x(2); \]
\[ t = x(3); \]

%Input: m.file for determining cross-sectional area for specific geometry.
%Output: cross-sectional area of the links.
\[ [A_c] = A_I \text{beam sup}(h, w, t); \]
%Input: m.file containing constants of robotic arm under examination.
%Output: values of constants necessary to perform the following calculations.
[L, g, rho, m\_motor, W, M\_max, eff, v, E] = constants;

%Non-linear inequality constraint based on ensuring that the moment of the arm
doesn’t exceed the amount that the motor can safely withstand.
c = [Ac+((2*\((m\_motor/2) + W)\)/\(L\*rho\))-((2*M\_max*eff)/(g\*rho\*L\^2))];

%Non-linear equality constraints.
ceq = [];

### A.4 Area of I-beam Supported Rectangular Link

%Program: A\_I\_beam\_sup.m
%Determines the cross-sectional area (m\^2) for a rectangular cross-section with an
I-beam support structure. Using the user inputs of height, width, and thickness it
outputs a value for the variable Ac.

function Ac = A\_I\_beam\_sup(h, w, t)

Ac = \(2\*w\*t + 3\*(h-2\*t)\*t\);

### A.5 Inertia of I-beam Supported Rectangular Link

%Program: I\_I\_beam\_sup.m
%Determines the moment of inertia (m\^4) for a rectangular cross-section with an I-
beam support structure. Using the user inputs of height, width, and thickness it
outputs a value for the variable I.

function I = I\_I\_beam\_sup(h, w, t)
\[ I = \left( \frac{w \cdot h^3}{12} \right) - \left( \frac{((w-3t)/2) \cdot (h-2t) \cdot h^3}{6} \right); \]

### A.6 Area of X-Supported Rectangular Link

%Program: A_X_sup.m
% Determines the cross-sectional area (m²) for a rectangular cross-section with an X support structure. Using the user inputs of weight, width, and thickness it outputs a value for the variable Ac.

function Ac = A_X_sup(h, w, t)

\[ Ac = w \cdot h - \left( \frac{w-(2+(1/(\cosd(\atan(w/h)))))*t}{2} \right) \cdot \left( \frac{h}{2} \cdot \left( \frac{\cosd(\atan(h/w)-\atan(w/h))}{\cosd(\atan(h/w)-\atan(w/h))} \right)^3 \right) \]

### A.7 Inertia of X-Supported Rectangular Link

%Program: I_X_sup.m
% Determines the moment of inertia (m⁴) for a rectangular cross-section with an X support structure. Using the user inputs of height, width, and thickness it outputs a value for the variable I.

function I = I_X_sup(h, w, t)

\[ I = \left( \frac{w \cdot h^3}{12} \right) - \left( \frac{2 \cdot ((w-(2+(1/(\cosd(\atan(w/h)))))*t) \cdot h}{2} \cdot \left( \frac{\cosd(\atan(h/w)-\atan(w/h))}{\cosd(\atan(h/w)-\atan(w/h))} \right)^3 \right) \cdot \left( \frac{1}{36} \right) \]

\[ + \left( \frac{w-(2+(1/(\cosd(\atan(h/w)))))*t}{2} \right) \cdot \left( \frac{h}{2} \cdot \left( \frac{\cosd(\atan(h/w)-\atan(w/h))}{\cosd(\atan(h/w)-\atan(w/h))} \right)^3 \right) \]
\[
\begin{align*}
&\left(\cos(d(\arctan(h/w) - \arctan(w/h))))\right)\left(t/3\right) + t\right) + 2) \right) / 2)
- \left(2*\left((h-2+\left(1/\left(\cos(d(\arctan(h/w))))\right)\right)\right)\right)3^{*} \\
&\left(w/2-\left(1+\left(\sin(d(\arctan(w/h))/\left(\cos(d(\arctan(h/w)-\arctan(w/h))))\right)\right)\right)\right) / 48));
\end{align*}
\]

A.8 Linear Constraints

%Program: A_lincon.m
%The matrix A developed from the left hand side of the equations from the two linear inequality constraints.
function A = A_lincon

A = [0, 1, -4, 1, 1, 1; 1, -1, 0, 1, 1, 1]

%Program: b_lincon.m
%The vector b developed from the right hand side of the equations from the two linear inequality constraints.
function b = b_lincon

b = [0; 0]

A.9 Output for I-beam Supported Rectangular Link

>> x0 = [0.7, 0.5, 0.003]; % Make a starting guess at the solution
options = optimset('Largescale','off');
[x,fval,exitflag] = ...
fmincon(@objfun,x0,[0,-1,4 ;-1,1,0],[0;0],[],[],[0.045, 0.045, 0.002],[0.14, 0.14, 0.01],
@nonlincon,options)
Optimization terminated: first-order optimality measure less than options.TolFun and
maximum constraint violation is less than options.TolCon.

Active inequalities (to within options.TolCon = 1e-006):

<table>
<thead>
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<th>upper</th>
<th>ineqlin</th>
<th>ineqnonlin</th>
</tr>
</thead>
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<td>1</td>
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<td></td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

\[ x = \]

\[
  0.1400 \quad 0.1400 \quad 0.0024
\]

\[ fval = \]

\[
  3.7146e-004
\]

\[ exitflag = \]

\[
  1
\]

\[ >> x(2)-x(1) \]

\[ ans = \]

\[
  0
\]

\[ >> 4*x(3)-x(2) \]

\[ ans = \]

\[
  -0.1305
\]

\[ >> [c] = nonlincon(x) \]

\[ c = \]
A.10 Output for X-Supported Rectangular Link

>> x0 = [0.03, 0.02, 0.0045]; % Make a starting guess at the solution
options = optimset('Largescale','off');
[x,fval,exitflag] = ...
fmincon(@objfun,x0,[0,-1,4;-1,1,0],[0;0],[],[],[0.045, 0.045, 0.002],[0.14, 0.14,0.01],
@nonlincon,options)

Optimization terminated: first-order optimality measure less than options.TolFun and
maximum constraint violation is less than options.TolCon.
Active inequalities (to within options.TolCon = 1e-006):

<table>
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<th>ineqnonlin</th>
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</thead>
<tbody>
<tr>
<td>3</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

x =

0.1400 0.1052 0.0020

fval =

4.4736e-004

exitflag =

1

>> x(2)-x(1)

ans =
A.11 Output for I-beam Supported Rectangular Link Without Upper Bounds

>> x0 = [0.03, 0.02, 0.0045]; % Make a starting guess at the solution
options = optimset('Largescale','off');
[x,fval,exitflag] = ...
fmincon(@objfun,x0,[0,-1,4;-1,1,0],[0;0],[],[],[0.045, 0.045, 0.002],[1, 1,0.01],
@nonlincon,options)

Optimization terminated: magnitude of directional derivative in search direction less than 2*options.TolFun and maximum constraint violation is less than options.TolCon.
Active inequalities (to within options.TolCon = 1e-006):

lower  upper     ineqlin ineqnonlin
   3          1

x =
fval =

2.0192e-004

exitflag =

5

>> x(2)-x(1)

ans =

-0.16926

>> 4*x(3)-x(2)

ans =

-0.05624

>> [c] = nonlincon(x)

c =

-7.2891e-006

### A.12 Output for X-Supported Rectangular Link Without Upper Bounds

>> x0 = [0.4, 0.1, 0.002]; % Make a starting guess at the solution

options = optimset(‘Largescale’,’off’);
[x,fval,exitflag] = ...

fmincon(@(objfun,x0,[0,-1,4;-1,1,0],[0;0],[],[],[0.045, 0.045, 0.002],[1, 1,0.01],
     @nonlincon,options)

Optimization terminated: magnitude of directional derivative in search direction less
than 2*options.TolFun and maximum constraint violation is less than options.TolCon.
Active inequalities (to within options.TolCon = 1e-006):

<table>
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</tr>
</tbody>
</table>

x =

0.18479  
0.049934 
0.0020

fval =

3.4535e-004

exitflag =

5

>> x(2)-x(1)

ans =

-0.13486

>> 4*x(3)-x(2)

ans =

-0.04193
>> [c] = nonlincon(x)

c =

-1.9566e-007
Appendix B

Danaher Data Sheets

B.1 Motors
### B/M MOTORS

#### B/M 80X SERIES MOTORS

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of Poles</td>
<td></td>
<td></td>
</tr>
<tr>
<td>DC Res at 25°C</td>
<td>Rm</td>
<td>ohms</td>
</tr>
<tr>
<td>Max Theoretical Acceleration</td>
<td>Z</td>
<td>rad/sec²</td>
</tr>
<tr>
<td>Motor Constant at 25°C</td>
<td>Kb</td>
<td>VRMS / krpm</td>
</tr>
<tr>
<td>Peak Torque</td>
<td>Tps</td>
<td>N·m (lb-ft)</td>
</tr>
<tr>
<td>Cont. Line Current</td>
<td>Ics</td>
<td>ARMS</td>
</tr>
<tr>
<td>Peak Line Current</td>
<td>Ips</td>
<td>ARMS</td>
</tr>
<tr>
<td>Kilowatts</td>
<td>kW</td>
<td></td>
</tr>
<tr>
<td>Torque Sensitivity (Stall) ±10%</td>
<td>Kt</td>
<td></td>
</tr>
<tr>
<td>NMAX rpm</td>
<td>RMAX</td>
<td>rpm</td>
</tr>
<tr>
<td>Thermal Resistance at Stall</td>
<td>Rth</td>
<td>°C/watt</td>
</tr>
<tr>
<td>Inductance (Line-to-Line) ±30%</td>
<td>Lm</td>
<td>mH</td>
</tr>
<tr>
<td>Kilowatts</td>
<td>kW</td>
<td></td>
</tr>
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<td>Torque Sensitivity (Stall) ±10%</td>
<td>Kt</td>
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<td>Rth</td>
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</tr>
<tr>
<td>Inductance (Line-to-Line) ±30%</td>
<td>Lm</td>
<td>mH</td>
</tr>
</tbody>
</table>

#### B/M 40X & 60X SERIES MOTORS

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>Units</th>
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</thead>
<tbody>
<tr>
<td>Number of Poles</td>
<td></td>
<td></td>
</tr>
<tr>
<td>DC Res at 25°C</td>
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<td>rad/sec²</td>
</tr>
<tr>
<td>Motor Constant at 25°C</td>
<td>Kb</td>
<td>VRMS / krpm</td>
</tr>
<tr>
<td>Peak Torque</td>
<td>Tps</td>
<td>N·m (lb-ft)</td>
</tr>
<tr>
<td>Cont. Line Current</td>
<td>Ics</td>
<td>ARMS</td>
</tr>
<tr>
<td>Peak Line Current</td>
<td>Ips</td>
<td>ARMS</td>
</tr>
<tr>
<td>Kilowatts</td>
<td>kW</td>
<td></td>
</tr>
<tr>
<td>Torque Sensitivity (Stall) ±10%</td>
<td>Kt</td>
<td></td>
</tr>
<tr>
<td>NMAX rpm</td>
<td>RMAX</td>
<td>rpm</td>
</tr>
<tr>
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<td>Rth</td>
<td>°C/watt</td>
</tr>
<tr>
<td>Inductance (Line-to-Line) ±30%</td>
<td>Lm</td>
<td>mH</td>
</tr>
</tbody>
</table>

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### Parameter Symbol and Units

- **EB-602-C**
- **EB-604-A**
- **EB-604-B**
- **EB-604-C**
- **EB-606-A**
- **EB-606-B**
- **EB-606-C**
- **EB-606-D**
- **EB-802-A**

---

### Raw Text

```
<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>Kilowatts</td>
<td>kW</td>
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<td>Rth</td>
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</tr>
<tr>
<td>Inductance (Line-to-Line) ±30%</td>
<td>Lm</td>
<td>mH</td>
</tr>
</tbody>
</table>
```
Notes:
1. BE and ME outline and dimension data and connector information is available by contacting the Kollmorgen Customer Support Network.
2. Dimensions in mm (inches)
   Tolerances, unless otherwise specified:
   metric: X decimal place ±.4, XX decimal places ±.13
   inches: XX decimal places ±.015, XXX decimal places ±.005

<table>
<thead>
<tr>
<th></th>
<th>A</th>
<th>B</th>
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</thead>
<tbody>
<tr>
<td>B-802</td>
<td>360.4 (14.19)</td>
<td>300.5 (11.83)</td>
</tr>
<tr>
<td>B-804</td>
<td>449.9 (17.71)</td>
<td>390.0 (15.35)</td>
</tr>
<tr>
<td>B-806</td>
<td>539.4 (21.24)</td>
<td>479.5 (18.88)</td>
</tr>
<tr>
<td>B-808</td>
<td>628.9 (24.76)</td>
<td>569.0 (22.40)</td>
</tr>
<tr>
<td>M-803</td>
<td>449.9 (17.71)</td>
<td>390.0 (15.35)</td>
</tr>
<tr>
<td>M-805</td>
<td>539.4 (21.24)</td>
<td>479.5 (18.88)</td>
</tr>
<tr>
<td>M-807</td>
<td>648.7 (25.54)</td>
<td>588.8 (23.18)</td>
</tr>
</tbody>
</table>
B.2 Gearheads
DuraTRUE 90™ Size 90

Right Angle Gearheads

All dimensions are: mm (inch)

AD** = Adapter length

Adapter length will vary depending on motor. Efficiency is calculated at 100% of the rated torque.

Performance Specifications

<table>
<thead>
<tr>
<th>Part Number</th>
<th>Ratio1</th>
<th>Dimension 'K' mm [in]</th>
<th>Dimension 'L' mm [in]</th>
<th>Backlash [arc-min]</th>
<th>Weight kg [lb]</th>
<th>Efficiency</th>
</tr>
</thead>
</table>

1 Ratios are exact; higher ratios and other custom options are also available, consult factory.

Tpeak = Allowable momentary peak torque for emergency stop or heavy shock loading.

J = Mass moment of inertia reflected to the input shaft (including pinion assembly).

Efficiency is calculated at 100% of the rated torque.

5:1 to 50:1

1 Ratios are exact, higher ratios and other custom options are also available, consult factory.

Tpeak = Allowable momentary peak torque for emergency stop or heavy shock loading.

J = Mass moment of inertia reflected to the input shaft (including pinion assembly).

5:1 to 50:1

Tpeak = Allowable momentary peak torque for emergency stop or heavy shock loading.

J = Mass moment of inertia reflected to the input shaft (including pinion assembly).
### Performance Specifications

<table>
<thead>
<tr>
<th>Part Number</th>
<th>Ratio</th>
<th>Dimension 'K' (mm [in])</th>
<th>Dimension 'L' (mm [in])</th>
<th>Backlash (arc-min)</th>
<th>Weight (kg [lb])</th>
<th>Efficiency</th>
</tr>
</thead>
<tbody>
<tr>
<td>DTR60-005</td>
<td>5:1</td>
<td>79 [3.11]</td>
<td>109.5 [4.31]</td>
<td>9 max</td>
<td>2.5 [5.5]</td>
<td>93%</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Part Number</th>
<th>Ratio</th>
<th>Dimension 'K' (mm [in])</th>
<th>Dimension 'L' (mm [in])</th>
<th>Backlash (arc-min)</th>
<th>Weight (kg [lb])</th>
<th>Efficiency</th>
</tr>
</thead>
<tbody>
<tr>
<td>DTR60-009</td>
<td>9:1</td>
<td>25 [0.98]</td>
<td>16 [0.63]</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>DTR60-010</td>
<td>10:1</td>
<td>22 [0.87]</td>
<td>15 [0.59]</td>
<td></td>
<td></td>
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</tr>
<tr>
<td>DTR60-012</td>
<td>12:1</td>
<td>20 [0.79]</td>
<td>16 [0.64]</td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>DTR60-015</td>
<td>15:1</td>
<td>17 [0.67]</td>
<td>15 [0.59]</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>DTR60-020</td>
<td>20:1</td>
<td>15 [0.59]</td>
<td>20 [0.79]</td>
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<tr>
<td>DTR60-025</td>
<td>25:1</td>
<td>13 [0.51]</td>
<td>25 [0.99]</td>
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<tr>
<td>DTR60-030</td>
<td>30:1</td>
<td>12 [0.47]</td>
<td>30 [1.18]</td>
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<td></td>
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<tr>
<td>DTR60-040</td>
<td>40:1</td>
<td>11 [0.43]</td>
<td>40 [1.57]</td>
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<tr>
<td>DTR60-050</td>
<td>50:1</td>
<td>10 [0.39]</td>
<td>50 [1.97]</td>
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<tr>
<td>DTR60-060</td>
<td>60:1</td>
<td>9 [0.35]</td>
<td>60 [2.36]</td>
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<td></td>
</tr>
<tr>
<td>DTR60-075</td>
<td>75:1</td>
<td>8 [0.31]</td>
<td>75 [2.95]</td>
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<tr>
<td>DTR60-090</td>
<td>90:1</td>
<td>7 [0.28]</td>
<td>90 [3.54]</td>
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<tr>
<td>DTR60-100</td>
<td>100:1</td>
<td>6 [0.24]</td>
<td>100 [3.94]</td>
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</tr>
<tr>
<td>DTR60-120</td>
<td>120:1</td>
<td>5 [0.20]</td>
<td>120 [4.72]</td>
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<tr>
<td>DTR60-125</td>
<td>125:1</td>
<td>4 [0.16]</td>
<td>125 [4.92]</td>
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</tr>
<tr>
<td>DTR60-150</td>
<td>150:1</td>
<td>3 [0.12]</td>
<td>150 [5.91]</td>
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<td></td>
<td></td>
</tr>
<tr>
<td>DTR60-200</td>
<td>200:1</td>
<td>2 [0.08]</td>
<td>200 [7.88]</td>
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<tr>
<td>DTR60-250</td>
<td>250:1</td>
<td>1 [0.04]</td>
<td>250 [9.85]</td>
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<td></td>
</tr>
<tr>
<td>DTR60-300</td>
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<td>0.5 [0.02]</td>
<td>300 [11.81]</td>
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<td></td>
<td></td>
</tr>
<tr>
<td>DTR60-400</td>
<td>400:1</td>
<td>0.33 [0.01]</td>
<td>400 [15.75]</td>
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<tr>
<td>DTR60-500</td>
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<td>0.25 [0.01]</td>
<td>500 [19.71]</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

1 Ratios are exact, higher ratios and other custom options are also available, consult factory.

T<sub>r</sub> = Rated output torque at rated speed for specific hours of life.

T<sub>peak</sub> = Allowable momentary peak torque for emergency stop or heavy shock loading.

J = Mass moment of inertia reflected to the input shaft (including pinion assembly).